Student ID Number: __________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, 3 January 2008, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part B, 3 January 2008, 14:00–17:00

B1. The bound-state energy levels of the hydrogen atom are $E_n = -\mathrm{Ry}/n^2$ when one includes only the kinetic energy and the Coulomb attraction to the nucleus in the Hamiltonian. Here, Ry is a Rydberg which is approximately 13.6 eV.

(a) (*2 points*) What are the allowed values of n? What is the degeneracy of the $n = 2$ level, including the spin degeneracy?

(b) (*4 points*) Include a spin-orbit coupling term in the Hamiltonian of the form

$$\langle n', l', m_l', m_s' | H_1 | n, l, m_l, m_s \rangle = \frac{E_{so}}{\hbar^2} \langle l', m_l', m_s' | \vec{L} \cdot \vec{S} | l, m_l, m_s \rangle$$

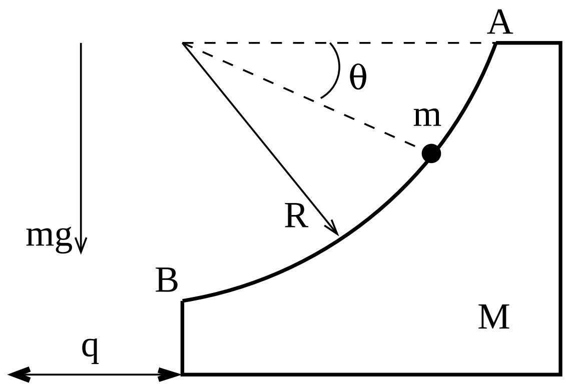
Here, $\vec{L}$ is the orbital angular momentum, and $\vec{S}$ is the spin of the electron. As is conventional, the orbital angular momentum quantum number is l, the L_z quantum number is m_l, and the S_z quantum number is m_s. (The true spin-orbit coupling would have E_{so} depend on n and l, but this form is more tractable.)

To linear order in E_{so}, what are the eigenvalues of the $n = 2$ level and their degeneracies?

Hint: use eigenstates of total angular momentum.

(c) (*4 points*) Also to linear order E_{so} what are the eigenvectors of the $n = 2$ level? Express the eigenvectors in terms of the basis states $|n, l, m_l, m_s\rangle$, where $n = 2$, l is the orbital angular momentum quantum number, m_l is the L_z orbital angular momentum quantum number, and m_s is the S_z spin quantum number.

B2. A point particle of mass m slides (without friction) down a wedge of mass M (see figure).



The wedge slides on a frictionless surface. The inner surface of the wedge is shaped as a cylinder of radius R. The initial position of mass m is at point A. Both the particle and the wedge start to move from rest.

(a) (*2 points*) Construct a Lagrangian for the entire system in terms of the angle θ and the displacement of the wedge, q.

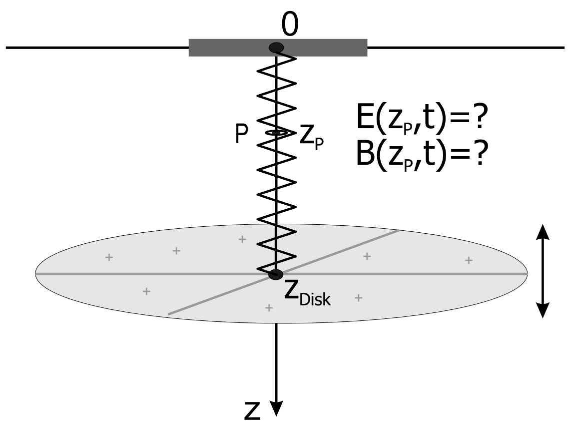
(b) (*2 points*) Derive the equations of motion.

(c) (*3 points*) Using energy and momentum conservation, find the speed of the particle relative to the wedge as a function of the angle θ.

(d) (*3 point*) For this part only, assume that the wedge is much heavier than the particle. Under this assumption, find the time it takes the particle to slide from A to B. [You may use without proof the fact that $\int_0^{\pi/2} dx/\sqrt{\sin(x)} = \sqrt{2}K\left(\sqrt{2}/2\right) \approx 2.62\ldots$, where $K(x)$ is the elliptic integral.]

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part B, 3 January 2008, 14:00–17:00

B3. A homogeneously charged circular disk with radius $R = 10$ cm and charge $Q = 0.3$ C is attached at one end of a dielectric, non-magnetic spring ($\epsilon = \epsilon_0, \mu = \mu_0$), shown as a zig-zag line in the figure, which makes the disc oscillate along the z axis. The surface of



the disk is perpendicular to the z axis. The position of the center of the disk is $z_{disk}(t) = z_{eq} + z_o \cos(2\pi f t)$, where $z_{eq} = 10$ cm is the equilibrium position of the disk, $z_o = 1$ cm is the amplitude and $f = 7$ Hz is the frequency of the disk oscillations.

(a) (*4 points*) Derive an expression for the electric field E at a point P(z_P) on the axis of the disc as a function of time.

Hint: find the static electric field $E(z)$ along the z axis, created by the charged disk when at rest. Neglect retardation effects.

(b) (*5 points*) Obtain an expression for the magnetic field B at the vicinity of point P(z_P) as a function of time. As an approximation of the magnetic field at point P, consider the magnetic field through a small loop with radius r, centered at point P, and parallel to the charged disk.

(c) (*1 point*) Calculate the amplitude of the magnetic field at point P if $z_P = 7$ cm and $r = 1$ mm.