Student ID Number: __________

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA

Part C, 4 January 2008, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. Consider a triangular molecule where each site is occupied by a single electron with spin $s = 1/2$. The electrons interact via an exchange interaction. The Hamiltonian for this system is given by

$$H = J(\vec{s}_1 \cdot \vec{s}_2 + \vec{s}_2 \cdot \vec{s}_3 + \vec{s}_3 \cdot \vec{s}_1)$$

where $J > 0$.

(a) (6 *points*) Calculate the magnetic energy levels, their spin quantum number and their degeneracy. Express the energies in terms of J.

Hint: the problem is most easily solved by expressing the Hamiltonian in terms of the total spin of the system.

(b) (4 *points*) Now consider applying a magnetic field $\vec{B}$ [add to the Hamiltonian $-g\mu_B(\vec{s}_1 + \vec{s}_2 + \vec{s}_3) \cdot \vec{B}$ where g is the Landé g-factor and μ_B is the Bohr magneton]. What are the energy levels of the molecule as a function of the magnetic field? Does the ground state (the lowest energy level) change at some magnitude of the field B^*? What are these states? Calculate $g\mu_B B^*/J$.

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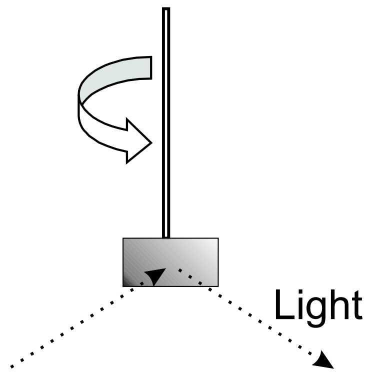
C2. A mirror galvanometer is a mirror attached to a long, fine wire that hangs vertically (see figure). By reflecting a light beam from the mirror, one can detect small thermal fluctuations



in the torsional angle θ of the wire. The energy of the mirror galvanometer is

$$E = \frac{1}{2} I\omega^2 + \frac{1}{2} B\theta^2$$

where I is the moment of inertia, B is a torsion constant, and $\omega = d\theta/dt$.

(a) (*4 points*) Suppose that measurements will be made at $T = 300$ K, and the experimenter requires the root-mean-squared fluctuations θ_{rms} to be at least 10^{-4} radians in magnitude. Then what should be the torsional constant B of the mirror galvanometer? Recall $k_B = 1.38065 \times 10^{-23}$ J/K.

(b) (*6 points*) Now suppose that we design a very unusual mirror galvanometer, where the energy is

$$E = \frac{1}{2} I\omega^2 + B\theta^4.$$

Find an expression for the mean-squared displacement $\langle \theta^2 \rangle$ at temperature T.

Potentially useful integrals:

$$\int_{-\infty}^{\infty} e^{-u^4} u^2 \, du \approx 0.613$$

$$\int_{-\infty}^{\infty} e^{-u^4} \, du \approx 1.82$$

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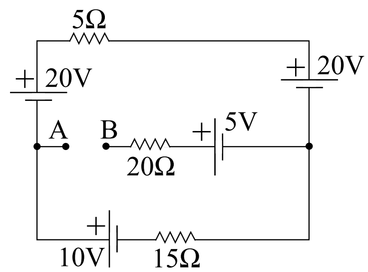
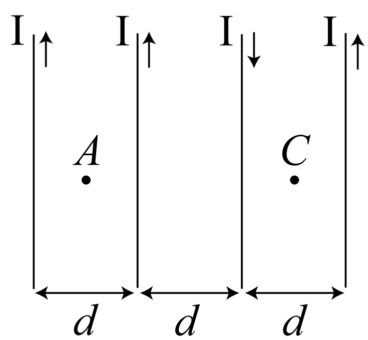
C3. (a) (*2 points*) What is the voltage difference $V_B - V_A$ (in V) in the circuit shown?

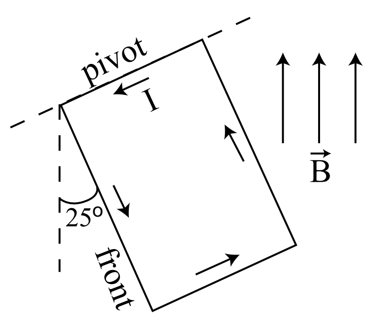


(b) (*2 points*) Four very long parallel current-carrying wires lie in a plane as shown. Their



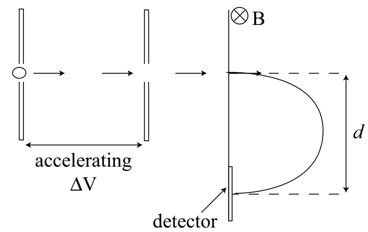
spacing is constant and the currents have the same values with their directions as indicated. What is the ratio of the magnetic fields $|B_C/B_A|$, where points A and C lie halfway between the adjacent wires?

(c) (*2 points*) A square conducting loop of sides 40 cm can pivot about its top side. A



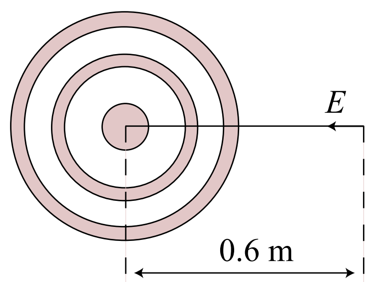
current I = 5 A flows through the loop which is placed in a uniform vertical magnetic field of strength 0.3 T. The loop makes an angle of 25 degrees with the vertical. What is the magnetic torque (in N· m) on the loop?

(d) (*2 points*) Two particles, H and S, are accelerated from rest by a common electrical potential before entering the bending region of a mass spectrometer (in which there is a magnetic field of B = 5.4 T). Particle S has a mass $m_s = 7.2 \times 10^{-26}$ kg and a charge



$q_s = 6.8 \times 10^{-18}$ C. Particles H and S hit the detector a distance d_h and d_s from the entrance slit, respectively, where $d_h/d_s = 1.05$. If the charge of H is 1.63×10^{-19} C, what is its mass (in kg)?

(e) (*2 points*) In the arrangement of conductors in the figure, the central sphere carries a net charge of $+5$ nC, the outer spherical shell carries a net charge of -10 nC. At a



radius of 0.6 m the electric field is directed inward and has a magnitude of 200 N/C. What is the net charge (in nC) on the inner shell?