Prelim ID Number: ______

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, 14:00–17:00, Jan 3, 2011

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work **different problems on separate sheets of paper**. The sheets for each problem will be stapled together but separately from the other two problems.
4. You will be assigned a **Prelim ID Number**, *different from your UF ID Number*. The **Prelim ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of **each sheet**. Do **NOT** use your name or UF ID Number anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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B1. Consider the electric field vector $\vec{E}(z)$ at a height z above the center of a square sheet (side length a) carrying a uniform surface charge density σ.

(a) (*3 points*) What must be the form of $\vec{E}(z)$ on the basis of dimensional analysis? Remember to include an arbitrary function of any dimensionless parameters you can make, and recall that the dimensions of the electric permittivity of free space are $\epsilon_0 \sim \text{charge}^2\,\text{time}^2\,\text{mass}^{-1}\,\text{length}^{-3}$.

(b) (*2 points*) What is the limiting form of $\vec{E}$ for $z \ll a$? Note that no computation is necessary for this.

(c) (*2 points*) What is the limiting form of $\vec{E}$ for $z \gg a$? Note that no computation is necessary for this.

(d) (*3 points*) Express $\vec{E}(z)$ as an integral over the charged square but do not attempt to evaluate the integral.

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B2. Consider an air conditioned room with a refrigerator inside. The temperature in the refrigerator is maintained at T_1, in the room the temperature is maintained at T_2, and the outside temperature is T_3. The temperatures are related by $T_1 < T_2 < T_3$. All heat produced by the refrigerator goes into the room and should be pumped out by the air conditioning unit to maintain the temperature T_2.

(a) (*6 points*) A massive object is brought from outside and placed inside the refrigerator. The object has heat capacity C. Calculate the total work W done by both the refrigerator and the air conditioner while cooling the object to T_1. Assume that both the refrigerator and the air conditioning unit are ideal heat machines.

(b) (*4 points*) Now consider another experiment. The same object is brought from the outside into the room. First it naturally cools to room temperature T_2. Next it is placed in the refrigerator and cooled to T_1. Will this require the same amount of work done by both the refrigerator and the air conditioner as in part (a)? If not, calculate the total work done. Which amount of work is smaller?

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B3. Let $|a\rangle$, $|b\rangle$, and $|c\rangle$, be three orthonormal states and let A be the operator

$$A = \alpha\, |b\rangle\langle c| + \beta\, |c\rangle\langle a| + \gamma\, |a\rangle\langle b| \tag{1}$$

where α, β, and γ are *complex numbers.*

(a) (*2 points*) What conditions must the complex numbers α, β, and γ satisfy for A to be unitary?

(b) (*2 points*) Compute the operator A^2, writing your answer in the same format as A above. (Note that you should be able to do this in your head).

(c) (*3 points*) What conditions must α, β, and γ satisfy for $A + A^2$ to be Hermitian? Assume that A is unitary as in part (a).

(d) (*3 points*) Compute A^3 while assuming that A is unitary and that $A + A^2$ is Hermitian. From your answer, what can you deduce about the possible eigenvalues of A?