Prelim ID Number: ______

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part D, 14:00–17:00, Jan 4, 2011

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work **different problems on separate sheets of paper**. The sheets for each problem will be stapled together but separately from the other two problems.
4. You will be assigned a **Prelim ID Number**, *different from your UF ID Number*. The **Prelim ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of **each sheet**. Do **NOT** use your name or UF ID Number anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part D, 14:00–17:00, Jan 4, 2011

D1. Some astronomical objects can be accurately modeled as consisting of a fluid of constant density that is spherical, nonrotating, and held together by its own gravity.

(a) (*3 points*) Use dimensional analysis to estimate the central pressure of such an object of mass M and radius R. Your answer might include other fundamental constants of nature such as Newton's gravitational constant G, the speed of light c, Planck's constant h, the charge of the electron e ...

(b) (*3 points*) For such an object, the pressure p of the fluid is a function of distance r away from the center. Find a differential equation for $dp(r)/dr$. You might find it convenient to use the constant density $\rho = M/(4\pi R^3/3)$ in your answer rather than the mass M. But, your answer must only include *either* ρ *or* M, but not both.

(c) (*3 points*) Solve this differential equation for the pressure $p(r)$ of the fluid.

(d) (*1 points*) The mass of the Earth is about $M = 6 \times 10^{24}$ kg, the radius of the Earth is about $R = 6 \times 10^6$ m. Estimate the pressure at the center of the Earth, and give your answer as a multiple of atmospheric pressure at the surface of the Earth, which is about 10^5 N/m^2.

For part (d) you may wish to know that, in MKS units

$$G = 6.67 \times 10^{-11}$$
$$c = 3 \times 10^{8}$$
$$h = 6.64 \times 10^{-34}$$
$$e = 1.6 \times 10^{-19} \ldots$$

DO NOT use these numerical values in any other part of this problem.

PRELIMINARY EXAMINATION
Department of Physics
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D2. Consider an electron with mass m_e confined within an infinite square-well potential defined by $V(x) = 0$ for $0 < x < L$ and $V(x) = +\infty$ otherwise.

(a) (*1 point*) Using Schrödinger's equation calculate the allowed stationary state eigenfunctions $\psi_n(x)$, where the complete wave functions are given by $\Psi_n(x,t) = \psi_n(x)\, e^{-iE_n t/\hbar}$. Normalize the eigenfunctions so that the probability of finding the electron somewhere in the square-well is one.

(b) (*2 points*) Show that the allowed energy levels of the system are, $E_n = E_0 n^2$, where $E_1 = E_0 = \hbar^2\pi^2/(2mL^2)$ is the ground state energy and $n = 1, 2, 3, \ldots$ Why is $n = 0$ excluded as a possible energy level?

(c) (*3 points*)
Consider the operator, $\mathcal{O} = (x)_{\rm op}(p_x)_{\rm op}$ (i.e. the product of the position operator times the momentum operator). Is $\mathcal{O}$ an hermitian operator? Calculate the expectation value of the operator $\mathcal{O}$ for the $n^{\rm th}$ stationary state (i.e. calculate $\langle\psi_n|\mathcal{O}|\psi_n\rangle$).

(d) (*4 points*)
Suppose the particle in this infinite square well has an initial wave function at $t = 0$ given by

$$\Psi(x,0) = Ax(L-x). \qquad (2)$$

What is the normalization factor A? If at a later time t you measure the energy of this particle, what is the probability, P_{E_1}, that you will measure the ground state energy $E_1 = E_0$? When you measure the energy at this same later time, what is the probability, P_{E_2}, that you will measure the first excited state energy E_2?

Helpful Integrals:

$$\int \sin x\, \cos x\, dx = \frac{1}{2}\sin^2 x$$

$$\int x\sin x\, dx = \sin x - x\cos x$$

$$\int x^2 \sin x\, dx = 2x\sin x - (x^2-2)\cos x$$

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$\int x\sin^2 x\, dx = \frac{x^2}{4} - \frac{x\,\sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int x^2\sin^2 x\, dx = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right) - \frac{x\cos 2x}{4}$$

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D3. Two identical balls each with mass M are attached to the ends of a thin rod of length L and negligible mass. The rod is free to rotate in a vertical plane without friction about a horizontal axis through its center. With the rod initially horizontal, a small wad of wet putty with mass m drops vertically onto one of the balls, hitting it with a speed of v and sticking to it.

(a) (*2 points*) What is the angular speed of the system just after the putty wad hits one of the masses M?

(b) (*2 points*) What is the ratio of the kinetic energy of the system after the collision to that of the putty wad just before?

(c) (*2 points*) Through what angle will the system rotate before it momentarily stops, assuming $v^2/gL < 1$?

(d) (*4 points*) After many oscillations, air resistance has taken away most of the energy. At that time what is the frequency of small oscillations when the ball with the wad of wet putty oscillates at the bottom?