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# PRELIMINARY EXAMINATION 

Department of Physics
University of Florida
Part B, January, 2012, 14:00-17:00

## Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."

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B1. A paramagnetic solid consists of $N$ non-interacting spins fixed at different sites of a lattice. The $z$ component of spin $i$ can take one of three values: $S_{i}^{z}=0$ or $\pm 1$, where $i=1 \rightarrow N$. In an external magnetic field $B$, this system is described by the Hamiltonian

$$
H=-\gamma B \sum_{i=1}^{N} S_{i}^{z}
$$

where $\gamma$ is a positive constant.
(a) (2 points) Calculate the canonical partition function at temperature $T$ for the case of a single spin $(N=1)$.
(b) (2 points) Calculate the canonical partition function at temperature $T$ for a general value of $N$.
(c) (2 points) Calculate the free-energy $F(T, B, N)$ of this system.
(d) (2 points) Calculate the magnetization $M(T, B, N)$ of this system.
(e) (2 points) Calculate the zero-field magnetic susceptibility $\chi(T, N)$ of this system.

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B2. (a) (3 points) A metal block with heat capacity $C$ is initially at temperature $T_{1}$. It is placed in contact with a heat reservoir held at temperature $T_{2}$. The block comes to the temperature of the reservoir through quasi-static heat flow without the performance of any work. What is the total entropy change of the universe, as a result of this process? Note that $T_{1}$ may be greater or less than $T_{2}$.
(b) (2 points) Define $x=T_{1} / T_{2}$. Are there any values of $x$ for which the total entropy change is negative? Prove it.
(c) (5 points) Now imagine that the same metal block (initially at temperature $T_{1}$ ) is connected to a reservoir (at temperature $T_{2}$ where $T_{2}<T_{1}$ ) by means of an ideal reversible heat engine (e.g. Carnot engine). The engine runs and produces work until the block attains the temperature of the reservoir. How much work $W$ is produced?

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## Lorentz Transformation Of Electromagnetic Fields

B3. Consider a cylindrical charge distribution. Let the charge density drop as a Gaussian distribution of the distance from the $z$ axis in the rest frame of the charge distribution (S):

$$
\rho(\vec{r})=A e^{-\left(x^{2}+y^{2}\right)}
$$

The distribution is passing by an observer, whose reference frame is $S^{\prime}$. In the rest frame of the observer, the charge distribution is moving at a velocity $\vec{v}=v_{z} \hat{z}$.
(a) (3 points) Compute the electric fields $\vec{E}$ and $\vec{B}$ in the rest frame of the charge distribution, $S$.
(b) (4 points) Compute the charge and current densities $\rho^{\prime}$ and $\vec{j}^{\prime}$ in the frame of the observer, $S^{\prime}$.
(c) (3 points) Compute the electric $\left(\overrightarrow{E^{\prime}}\right)$ and magnetic $\left(\overrightarrow{B^{\prime}}\right)$ fields in the frame $S^{\prime}$.

