Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, January, 2012, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

C1. A charged pion (π^+) is a particle that has a mass of 139 MeV/c^2 and decays into a muon (mass 106 MeV/c^2) and a (mass-less) neutrino.

(a) **(3 points)** Calculate the momentum (in MeV/c) and energy (in MeV/c^2) of the muon and the neutrino in the rest frame of the pion.

(b) **(4 points)** The half-life of the π^+ is 2.5×10^{-8}s. In a particular experiment, half a stream of mono-energetic pions decay before they cover a distance of 15 meters. Calculate the speed of the pions (in terms of the speed of light, $c = 3 \times 10^8$ m/s.)

(c) **(3 points)** Neutral pions (π^0)s have a mass of 135 MeV/c^2 and decay very quickly into two γ rays. When an energetic π^0 is produced, the energy of the decay products, in the lab frame, depends on the angle between the direction of the decay products in the center-of-mass frame, and the direction of the center-of-mass motion in the lab frame If the π^0 has $\beta = 0.5$. Using the Lorentz transform, $p = \gamma(p' + vE'/c^2)$, or otherwise, what are the maximum and minimum energies of the decay products in MeV/c^2? ($\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$)

C2. **(10 points)** What is the external magnetic field produced by a uniformly magnetized cylinder when $\vec{M}_0 = M_0\hat{x}$. Assume the cylinder extends infinitely along the z-axis and has a radius R. You may use any technique to generate the solution, but you must show all steps.

You might want to recall that the magnetic scalar potential, Φ_m, in cylindrical coordinates may be written as

$$\Phi_m(s, \phi) \;=\; a_o \;+\; b_o \ln s \;+\; \sum_{k=1}^{\infty} \left(a_k\, s^k \;+\; b_k\, s^{-k}\right) \left(c_k \cos k\phi \;+\; d_k \sin k\phi\right) \qquad .$$

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C3. Most of the stars in a spiral galaxy, such as our Milky Way, are located in a spherical bulge at the galaxy center. A small number of stars are in the spiral arms extending out from the bulge. (Our sun is located in one of these spiral arms.) See Fig. 1 for a sketch of the galaxy as seen from far away.

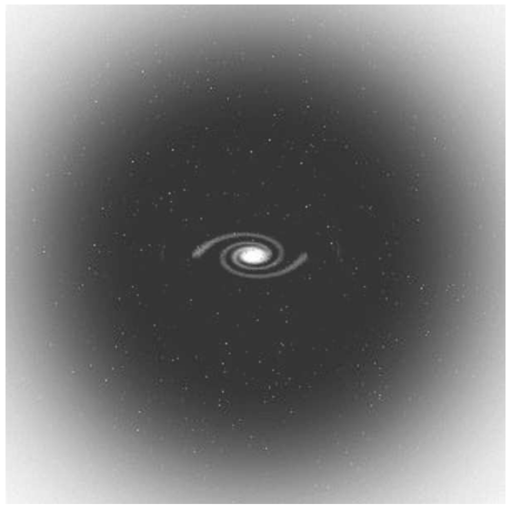

Fig. 1. A galaxy, viewed at a small angle away from directly above. The bulge is the bright sphere in the center, and two spiral arms extend out from the bulge. There is also a spherical "halo" or cloud of dark matter extending out from the bulge to a distance of 20× the radius of the bulge, well beyond the ends of the spiral arms.

Let us make a model of the orbital motion of these stars. Make five simplifying assumptions: (1) The bulge is a sphere. (2) The number of stars in the bulge is so much larger than the number in the arms that the stars in the bulge control the orbital dynamics. (3) The stars in the bulge are uniformly distributed, so that the bulge has density

$$\rho = \frac{3M}{4\pi R^3}$$

where M is the total mass of the galaxy and R the radius of the bulge. (4) All stars are in circular orbits about the common center of mass. (5) Ignore for the moment the halo shown in Fig. 1.

(a) **(2 points)** Calculate the velocity of the stars inside the bulge as a function of their radius from the center, $r \leq R$.

(b) **(2 points)** Calculate the velocity of the stars outside the bulge as a function of their radius from the center, $r \geq R$.

(c) **(2 points)** Make a sketch of the orbital velocities found in parts (a) and (b) as a function of orbit radius r. Use a scale of $0 \leq r \leq 9R$. What is the maximum value of the velocity, v_{max}? What is the value when $r = 9R$? What is the value when $r = 0$?

The actual plot of v versus r looks quite different than what you just plotted. A curve for one galaxy is shown in Fig. 2. The curve is quite flat for radii above 6 kpc, which we can take as the radius R of the bulge.

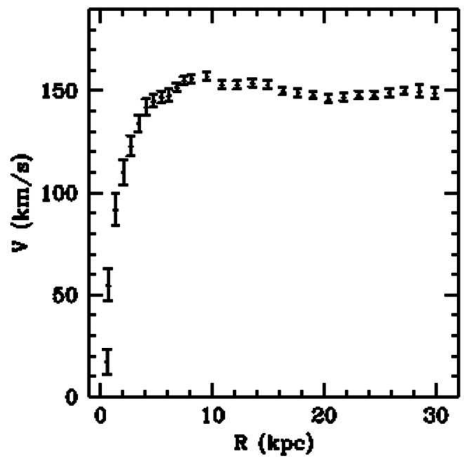

Fig. 2. Typical rotation curve (v versus r) of a galaxy.

We need to include the halo in our model in order to account for the flat portion of Fig. 2. Replace assumption (5) above with the following: (5) The spherical halo extends from the edge of the bulge, $r = R$, to $r = 20R$. (6) The halo density is *not* constant in r although it is constant as a function of angle at a given r. (7) The velocity is constant at v_{max} over $R \leq r \leq 20R$, where v_{max} is the velocity found in part (c) above.

(d) **(4 points)** Calculate the halo mass density outside the bulge as a function of distance r from the center. Here, $R \leq r \leq 20R$.