PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part D, January, 2012, 014:00–17:00

D1. A particle with energy E and mass m is in the one dimensional potential

$$\begin{aligned} V(x) &= \infty \text{ for } x < 0 \\ V(x) &= -V_0 \text{ for } 0 < x < a \\ V(x) &= 0 \text{ for } a < x. \end{aligned}$$

with $V_0 > 0$. In this problem we will find the bound states, $-V_0 < E < 0$.

(a) **(2 points)** Solve the one dimensional Schrodinger equation for ψ in the region $0 < x < a$. Given that $V(x) = \infty$ for $x < 0$, what is the general form of the solution in this region?

(b) **(2 points)** Solve the one dimensional Schrodinger equation in the region $x > a$. Given that we want the wave function to be normalizable, what is the general form of the solution in this region?

(c) **(2 points)** Match the boundary conditions at $r = a$ and derive a equation for the bound states.

(d) **(2 points)** Derive the condition on V_0 for there will not be a bound state for $-V_0 < E < 0$.

(e) **(2 points)** Derive the condition on V_0 for there to be only one bound state for $-V_0 < E < 0$.

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part D, January, 2012, 014:00–17:00

D2. Consider a harmonic oscillator which consists of a mass m and a spring with a spring constant k. The motion of the mass is restricted in the vertical direction (y-axis) by a frictionless guide going through the oscillator (see the figure below). A very light string of linear mass density ρ is attached directly to the mass and extends infinitely along the x-axis under constant tension T. If the spring is compressed from its initial equilibrium length and released, the mass oscillates and waves will propagate outward from the mass. Let $y(x,t)$ and $Y(t)$ denote the height of the string and the mass above the x-axis at time t, respectively. The connection point to the mass is specified as $x = 0$. Ignore the effect of the string *weight* (gravitational effect due to the string mass) in answering the following questions.

(a) Show that for small amplitude oscillations the equation of motion for the mass is given by

$$m\frac{d^2Y}{dt^2} + kY = T\left.\frac{\partial y}{\partial x}\right|_{x=0}.$$

(b) The waves propagating along the x-axis is governed by the wave equation

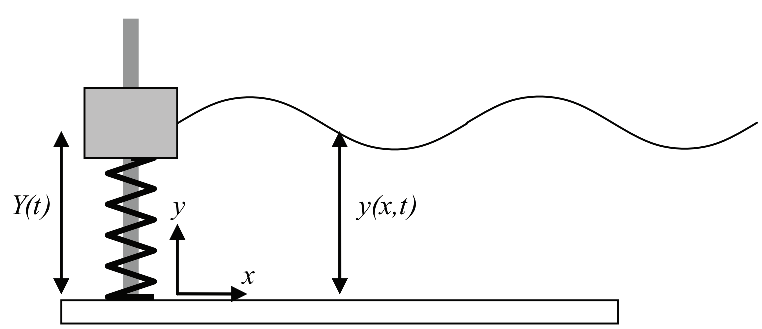
$$\left(\rho\frac{\partial^2}{\partial t^2} - T\frac{\partial^2}{\partial x^2}\right) y(x,t) = 0$$

Since the motion of the mass generates outgoing waves, the oscillator will experience damping (radiation damping), and the equation of motion will turn into the form of a damped harmonic oscillator:

$$m\frac{d^2Y}{dt^2} + \lambda\frac{dY}{dt} + kY = 0\,.$$

Express λ in terms of the quantities given in the problem. *Do not derive the wave equation.*

(c) What is $E(t)/E(0)$ for weak damping, $\lambda^2 \ll km$? $E(t)$ is the total mechanical energy of the oscillator at time t.



D3. An electron is at rest in an oscillating magnetic field along the z-direction:

$$\vec{b} = B_0 \cos(\omega t)\hat{z}$$

where B_0 and ω are constants.

(a) **(2 points)** Construct the Hamiltonian matrix for this system.

(b) **(4 points)** If at $t = 0$ the electron starts out in the spin-up state with respect to the x-axis (that is: $\chi(0) = \chi_+^{(x)}$), determine $\chi(t)$ at any subsequent time.

Beware: This is a *time-dependent* Hamiltonian, so you cannot get $\chi(t)$ in the usual way from stationary states. However, in this case you can solve the *time-dependent* Schröedinger equation directly.

(c) **(2 points)** Show that if you measure S_x, the probability of getting $-\hbar/2$ is: $\sin^2\left(\frac{\lambda B_0}{2\omega}\sin(\omega t)\right)$.

(d) **(2 points)** What is the minimum field (B_0) required to force a complete flip in S_x?

Hint: you may find the Pauli spin matrices useful. $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and remember that $S = \left(\frac{\hbar}{2}\right)\sigma$.