Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part C, January, 2014, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. An atom has total angular momentum $j = 1$. In the following we use the conventional notation that the state $|j, m\rangle$ has a total angular momentum (J^2) eigenvalue of $\hbar^2 j(j+1)$ and a J_z eigenvalue of $\hbar m$.

(a) [**2 points**] What are the matrix elements of the z-component of the angular momentum, $\langle 1, m|J_z|1, m'\rangle$? Express your answer as a matrix.

(b) [**2 points**] What are the matrix elements of the x-component of the angular momentum, $\langle 1, m|J_x|1, m'\rangle$? Express your answer as a matrix.

(c) [**2 points**] At $t = 0$ the x-component of the angular momentum is measured and found to be zero. What is the state of the system right after the measurement in terms of the $|1, m\rangle$ states?

(d) [**2 points**] For $t > 0$ the system evolves in time with the Hamiltonian $H = \omega L_z$. What is the state of the system at time t after the L_x measurement?

(e) [**2 points**] Another measurement of L_x is then made at time t. What is the probability of finding L_x to be zero at time t?

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C2. Electricity and Magnetism is governed by equations written in the 19th century by Maxwell. In this problem you will use these to show that plane electromagnetic waves are solutions to these equations, derive the dispersion relation of light, and relations amongst the fields.

(a) [**2 points**] Write Maxwell's equations for macroscopic media. Allow there to be some external charge ρ_{ext} and free current $\vec{\jmath}_{\text{free}}$. Define the quantities appearing in these equations.

(b) [**2 points**] What are the definitions of the auxiliary fields $\vec{D}$ and $\vec{H}$ which appear in these equations? Define any quantities appearing in these equations.

(c) [**2 points**] Let the medium to be electrically neutral, so that $\rho_{\text{ext}} = 0$. (Hint: it is composed of atoms, and may be a metal, so that $\vec{\jmath}_{\text{free}}$ is NOT zero.) Take each of the fields and currents to be of plane-wave form, so that the electric field is

$$\vec{E} = \vec{E}_0 e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

with corresponding plane waves for $\vec{D}$, $\vec{E}$, $\vec{H}$, $\vec{B}$, and $\vec{\jmath}$. Show that Maxwell's equations reduce to algebraic vector equations involving these fields, the wave vector $\vec{q}$, and the frequency ω.

(d) [**2 points**] The vector dot and cross products in the above tell us a lot about how the fields appear, but first let us simplify to vacuum. Let the current and the magnetic and electric dipole moments be zero: $\vec{\jmath} = 0$, $\vec{P} = 0$, $\vec{M} = 0$. Then eliminate the fields $\vec{D}$ and $\vec{B}$ and show that $\vec{q}$ $\vec{E}$ and $\vec{H}$ are orthogonal, and form a right-handed set. Find the *dispersion relation* between q and ω. Find also the relation between the electric and magnetic field strengths.

(e) [**2 points**] To consider matter, make five simplifications:
(1) Local response: Assume that the response is local, that the current at point $\vec{r}$ depends only on the electric field at that point.
(2) Non-magnetic materials: Then the magnetization is zero everywhere.
(3) Isotropic materials: You do not need to use tensors for the linear relations between fields and currents; scalars are sufficient.
(4) Linear materials: The currents are linear functions of fields.
(5) Homogeneous media: The materials properties do not depend on position in space.
With these assumptions, I can write

$$\vec{\jmath} = \sigma_1 \vec{E}$$

and

$$\vec{D} = \epsilon_1 \vec{E}.$$

where σ_1 is the conductivity and ϵ_1 is the dielectric constant. Substitute these into Maxwell's equations and show again that $\vec{q}$ $\vec{E}$ and $\vec{H}$ are orthogonal, and form a right-handed set. Find the *dispersion relation* between q and ω in terms of σ_1 and ϵ_1 .

Find also the relation between the electric and magnetic field strengths.

Hint: the dispersion relation is an equation of the form $q = f(\sigma_1, \epsilon_1, \omega)$ and your job is to find the functional form. q is the magnitude of $\vec{q}$.

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DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

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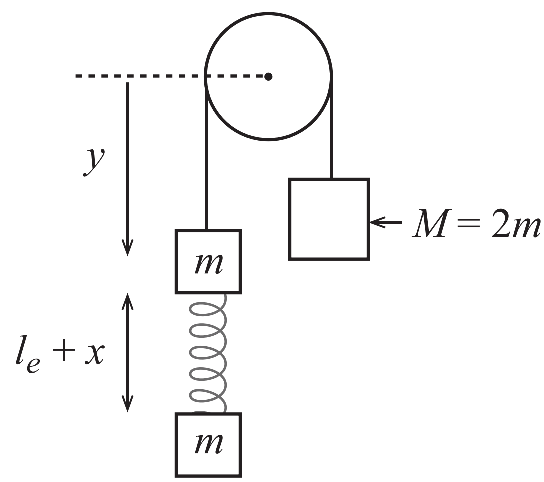
C3. Consider the modified Atwood machine as shown in the figure. Each weight on the left has mass m, and they are connected by a massless spring of force constant k. The weight on the right has mass $M = 2m$, and the pulley is massless and frictionless. The coordinate x is the extension of the spring from its equilibrium length; that is, the length of the spring is $l_e + x$ where l_e is the equilibrium length (with all weights in position and M held stationary).



(a) [**2 points**] Show that the total potential energy (spring plus gravitation) is just $U = \frac{1}{2}kx^2$ (plus a constant which we can take to be zero).

(b) [**3 points**] Find the canonical momenta conjugate to x and y. Solve for $\dot{x}$ and $\dot{y}$, and write down the Hamiltonian. Show that the coordinate y is ignorable.

(c) [**4 points**] Write down the four Hamiltonian equations and solve them for the following initial conditions. To begin, the mass M is held fixed such that $y = y_0$, and the lower mass m is pulled down a distance x_0. At $t = 0$ both masses are released from rest.
Hint: Write down the initial values of x and y and their momenta. You might solve the x equations by combining them into a second order equation for x. Once you know $x(t)$, you can quickly write down solutions for the other three variables.

(d) [**1 point**] Describe the motion, and identify the frequency with which x oscillates.