Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, January, 2017, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. **(Fry)** In models for electronic states in ring-shaped molecules, one is led to consider the following problem: Two identical fermions of mass m and spin $\frac{1}{2}$ are confined to a circular ring of radius r.

(a) [**3 points**] What are the energies and wavefunctions of single-particle states?

(b) [**4 points**] Find the energy, degeneracy, and wave functions for the ground state(s) and the first excited state(s) for two noninteracting particles.

(c) [**3 points**] The two particles of (b) interact via a potential of the form

$$V = C\cos(\phi_1 - \phi_2),$$

where ϕ_1 and ϕ_2 are the angular positions of the two particles, and the positive constant C is much smaller in magnitude than $\hbar^2/2mr^2$. Find the ground state and first excited state energies.

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A2. **(Thorn)** A non-relativistic particle of mass m, moving in one dimension, is incident from the left with energy E on a step potential given by $V(x) = 0$ for $x < 0$ and by $V(x) = V_0 > 0$ for $x > 0$. In the following parametrize the wave function by

$$\psi(x) = \begin{cases} e^{ikx} + Re^{-ikx} & x < 0 \\ Ae^{ik'x} + Be^{ik'x} & x > 0 \end{cases}$$

where $k > 0$.

(a) [**3 points**] For $E > V_0$, determine k and k' in terms of m, E, V_0, and, taking $k' > 0$, explain why $B = 0$ in the situation described above.

(b) [**1 points**] State the matching conditions on ψ and $d\psi/dx$ at $x = 0$.

(c) [**3 points**] Solve the matching conditions for $E > V_0$, to determine R in terms of.k, k', and calculate the probability that the particle is reflected by the step. Use conservation of probability to determine the transmission probability.

(d) [**1 point**] Explain why the transmission probability is *not* equal to $|A|^2$.

(e) [**2 points**] When $0 < E < V_0$, k' is no longer real. Determine it, and prove that in this case the particle will always be reflected by the step.

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DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

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A3. **(Konigsberg)** Two identical particles, fermions, each with mass M and spin $\frac{1}{2}$, are described by the Hamiltonian,

$$\hat{H} = -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + \frac{1}{2}M\omega^2(x_1^2 + x_2^2).$$

The two particles are in a state described by the wave function,

$$\psi(x_1, x_2) = \sqrt{\frac{2}{\pi}}\frac{(x_1 - x_2)}{a^2}\, e^{-(x_1^2+x_2^2)/2a^2}\, |\chi\rangle. \tag{1}$$

Here, $|\chi\rangle$ is a vector in a four-dimensional spin vector space, where $|m_1, m_2\rangle$ is a basis state for this space satisfying $\hat{S}_{jz}|m_1, m_2\rangle = m_j\hbar|m_1, m_2\rangle$ for $j = 1$ and 2, and $a = \sqrt{\hbar/M\omega}$.

(a) [**4 points**] What are the three mutually orthogonal and physically acceptable spin vectors $|\chi\rangle$ in the state described by Eq. (1)? Express your normalized answers in the basis $\{|m_1, m_2\rangle\}$.

(b) [**3 points**] Find the total energy of the system in the state described by Eq. (1).

(c) [**3 points**] Write down another stationary state of the system that is orthogonal to, but degenerate in energy with, the three states in part (a). Specify both the spatial and spin parts.