Student ID Number: ________

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part C, January, 2017, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. **(Whiting)** A bead of mass m is constrained to move on a hoop of radius $\mathbf{R}$. The hoop rotates with constant angular velocity ω around a diameter of the hoop, which is a vertical axis (line along which gravity acts).

(a) [**3 points**] Set up the Lagrangian for this system and obtain the equation of motion for the bead.

Let's use spherical polar coordinates with origin at the center of the hoop, and measure potential energy and $\theta = 0$ from the bottom of the hoop. Then the Lagrangian is given by:

$$\begin{aligned} L &= \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\sin^2\!\theta\,\omega^2 - mgR(1-\cos\theta), \quad \text{yielding:} \\ mR^2\ddot{\theta} &= mR^2\omega^2\sin\theta\cos\theta - mgR\sin\theta = mR^2\sin\theta\left(\omega^2\cos\theta - \frac{g}{R}\right). \end{aligned}$$

(b) [**4 points**] Find the critical angular velocity $\mathbf{\Omega}$ below which the bottom of the hoop provides a stable equilibrium for the bead.

To calculate the curvature of the effective potential, we need:

$$\frac{d}{d\theta}\left(\sin\theta\left[\omega^2\cos\theta - \frac{g}{R}\right]\right) = \omega^2\left(2\cos^2\theta - 1\right) - \frac{g}{R}\cos\theta.$$

Clearly, we can have $\ddot{\theta} = 0$, at $\sin\theta = 0$. Let the equilibrium occur at $\theta_0 = 0$. Then small departures from equilibrium are governed by:

$$\ddot{\delta\theta} = \left[\omega^2 - \frac{g}{R}\right]\delta\theta.$$

Thus, at $\theta_0 = 0$, the coefficient in brackets is negative (required for stability) if $\omega^2 < g/R$. Hence, $\Omega = \sqrt{g/R}$.

(c) [**3 points**] Find the stable equilibrium position for $\omega > \mathbf{\Omega}$.

For $\theta \neq 0$, the $\ddot{\theta} \neq 0$ unless $\theta = \arccos(g/R\omega^2)$. For this angle to exist, we require that $g/R\omega^2 < 1 \Rightarrow \omega > \sqrt{g/R} = \Omega$. Then, at $\theta_0 = \arccos(g/R\omega^2)$, the perturbation satisfies:

$$\ddot{\delta\theta} = \left(\frac{g^2}{R^2\omega^2} - \omega^2\right)\delta\theta,$$

and, since this is negative for $\omega > \Omega$, stable oscillations ensue.

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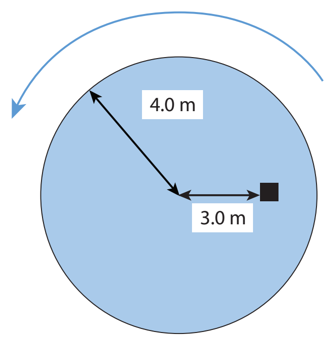
C2. **(Yelton)** A merry-go-round consists of a circular piece of wood of radius $R = 4$ meters. There is a box placed on the merry-go-round at a position $r = 3$ meters from the center.

The merry-go-round starts to rotate from rest with a an angular velocity given by $\omega = \beta t^2$ rad/s where β is a constant of value 0.25 rad/s^3 and t is in seconds.

(a) [**points**] At what time, T, is the centripetal acceleration of the box equal to its tangential acceleration?

(b) [**points**] At time of $t = 2$ seconds, the box begins to slide on the surface. What is the coefficient of static friction, μ_s, between the box and the merry-go-round?

(c) [**points**] On the figure below, if the position of the box shown is its position as it begins to slide, sketch the subsequent path of the box. As usual $0 < \mu_k < \mu_s$.



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C3. **(Stanton)** Consider a non-relativistic Fermi gas at absolute zero temperature, $\epsilon(k) = \hbar^2 k^2/2m$. The gas consists of N particles (spin-1/2) moving in a two-dimensional area A.

For $T = 0$:

(a) [**4 points**] Find expressions for both the Fermi wavevector k_F and the Fermi energy E_F for this gas, as functions of N and A.

(b) [**3 points**] Find the average kinetic energy per particle, $u = E/N$ and express it in terms of E_F.

(c) [**3 points**] What is the density of states (**in energy**) at the Fermi level $D(E_F)$? Give your answer in terms of E_F.