Student ID Number: ________

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part A, January, 2018, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

    (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

    (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

    (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

    (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

    (e) Each problem is worth 10 points.

    (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. **(Konigsberg)** A spin 1/2 electron with magnetic moment μ is placed in a uniform magnetic field $\vec{B} = B\hat{k}$ in the positive z-direction. The intrinsic spin of the electron is pointed along the positive x-direction at t=0.

(a) [**3 points**] Write down the Schrodinger equation for the two component wave function for the electron at rest

(b) [**3 points**] Find the corresponding time-dependent wave function

(c) [**2 points**] Calculate the expectation values $< S_x(t) >, < S_y(t) >, < S_z(t) >$

(d) [**2 points**] Find the probability as a function of time that the intrinsic spin will be pointed along:

- the positive z-direction
- the positive x-direction

This may be useful:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

$$| S_z^+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad | S_x^+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2}$$

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A2. **(Bartos)** A particle with mass m is allowed to move within

$$-\infty < x < \infty,$$
$$0 < y < a,$$
$$0 < z < b.$$

The particle is also exposed to a force $\mathbf{F} = (-kx{,}0{,}0)$.

(a) **[5 points]** Determine the energy levels of the particle's stationary states.

(b) **[5 points]** Determine the wave functions of the particle's stationary states.

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A3. **(Ingersent)** Consider a triangular molecule where each site is occupied by a single electron with spin $s = 1/2$. The electrons interact via an exchange interaction. The Hamiltonian for this system is given by

$$H = J(\vec{s_1} \cdot \vec{s_2} + \vec{s_2} \cdot \vec{s_3} + \vec{s_3} \cdot \vec{s_1})$$

where $J > 0$.

(a) [**6 points**] Calculate the magnetic energy levels, their total spin quantum number and their degeneracy. Express the energies in terms of J.
Hint: the problem is most easily solved by expressing the Hamiltonian in terms of the total spin of the system.

(b) [**4 points**] Now consider applying a magnetic field $\vec{B}$ [add to the Hamiltonian $-g\mu_B(\vec{s_1}+\vec{s_2}+\vec{s_3})\cdot\vec{B}$ where g is the Landé g-factor and μ_B is the Bohr magneton]. What are the energy levels of the molecule as a function of $B = |\vec{B}|$? The ground state (the lowest energy level) changes at $B = B^*$. What are the good quantum numbers of the ground states on either side of $B = B^*$? Calculate $g\mu_B B^*/J$.