Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, January 4, 2019, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. The eigenvectors and eigenvalues of a given Hamiltonian are

$$|\psi_a\rangle = \frac{1}{\sqrt{3}}\left(|1\rangle + |2\rangle + |3\rangle\right) \text{ with } E_a = 0 \quad (1)$$
$$|\psi_b\rangle = \frac{1}{\sqrt{6}}\left(|1\rangle - 2|2\rangle + |3\rangle\right) \text{ with } E_b = +E_o \quad (2)$$
$$|\psi_c\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle - |3\rangle\right) \text{ with } E_c = -E_o, \quad (3)$$

where $|1\rangle$, $|2\rangle$, $|3\rangle$ are an orthonormal basis of the system.

(a) **[3 points]** Suppose the state of the system is $\frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)$ right before an energy measurement is made. What are possible outcomes of the energy measurement and their associated probabilities?

(b) **[2 points]** Suppose the state of the system is at $t = 0$ is $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)$, but instead of making an energy measurement right away, the system evolves in time (without making an energy measurement). What is the state of the system at time t, $|\psi(t)\rangle$?

(c) **[3 points]** At time t a position measurement is made with the operator

$$X = 1|1\rangle\langle 1| + 2|2\rangle\langle 2| + 3|3\rangle\langle 3|. \quad (4)$$

What are the possible outcomes and their associated probabilities?

(d) **[2 points]** If steps (b) and (c) were repeated many times, what would the average value of the position measurements be?

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DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA

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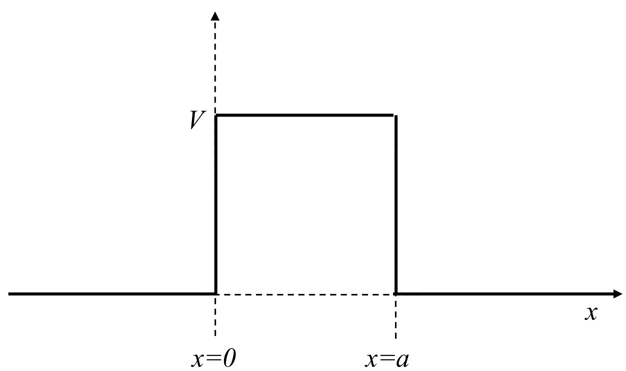
A2. A particle of mass m and energy E, moving in the positive direction along the x axis, impinges upon a potential barrier of height V and width a. Your goal is to calculate the transmission and reflection coefficients for the particle.



(a) [**1 point**] State the wave functions for the three domains on the x axis: $x < 0$, $0 < x < a$, and $x > a$, if $E > V$.

(b) [**1 point**] State the wave functions for the three domains on the x axis: $x < 0$, $0 < x < a$, and $x > a$, if $0 < E < V$.

(c) [**1 point**] State the conditions on the wave function at the boundaries $x = 0$ and $x = a$.

(d) [**7 points**] For $E > V$, compute the transmission and reflection probabilities for the particle. Express your answer in terms of E, V, and κa where $\kappa = \sqrt{2m(E-V)}/\hbar$. You may find the following expression useful:

$$(x+y)^4 + (x-y)^4 = 2(x^2-y^2)^2 + 16x^2y^2. \qquad (1)$$

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part A, January 4, 2019, 09:00–12:00

A3. Consider a quantum system in three dimensions.

(a) **[3 points]** Show that the Cartesian components of the angular momentum operator $\vec{L}$ do not commute with each other, and find the commutators $[L_x, L_y]$, $[L_x, L_z]$ and $[L_y, L_z]$.

(b) **[2 points]** Find the uncertainty product of the L_x and L_y observables, $\sigma_{L_x}\sigma_{L_y}$. You may use the generalized uncertainty principle for non-commuting observables:

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2 .$$

(c) **[4 points]** Show that the L^2 operator does commute with the L_x operator.

(d) **[1 point]** For a state with azimuthal quantum number $l = 3$, write down the possible values of the magnetic quantum number m.