Student ID Number: __________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, January 3, 2020, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. Consider a two-state system with stationary states $|\psi_1\rangle$ and $|\psi_2\rangle$ with energies E_1 and E_2. At $t = 0$ the system is described by

$$|\psi(0)\rangle = \frac{1}{2}[|\psi_1\rangle + |\psi_2\rangle]$$

(a) **[2 points]** Write down the state of the system $|\psi(t)\rangle$ at a later time t in terms of $|\psi_1\rangle$, $|\psi_2\rangle$, E_1 and E_2.

(b) **[8 points]** How long does it take for the system to evolve into a state orthogonal to the original one?

A2. Consider a Fermi surface, which is an equal-energy contour $E(\mathbf{k}) = \epsilon_F$ of a free electron in two dimensional (2d) momentum [$\mathbf{k} = (k_x, k_y)$] space.

(a) [**3 points**] Electrons have spin-1/2. Suppose the Hamiltonian of the electron is $\hat{H}(\mathbf{k}) = \mathbf{k}^2/(2m) + B\sigma_z$. Here and below σ_i is the Pauli matrix, given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

In this situation the Fermi surface splits into two. Obtain the equations of the Fermi surfaces in terms of ϵ_F, m, B.

(b) [**4 points**] For $\hat{H}(\mathbf{k}) = \mathbf{k}^2/(2m) + B\sigma_z + \lambda\mathbf{k}\cdot\vec{\sigma}$, obtain the equations of the Fermi surfaces.

(d) [**3 points**] For $\hat{H}(\mathbf{k}) = \mathbf{k}^2/(2m) + B\sigma_x + \lambda\mathbf{k}\cdot\vec{\sigma}$, obtain the equations of the Fermi surfaces. In the limit $\lambda \ll B, \epsilon_F$, sketch the shape of the Fermi surfaces.

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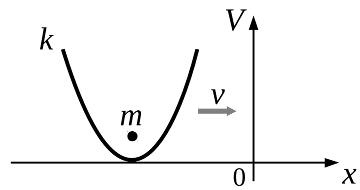
A3. Consider a quantum particle in a 1-D potential well. The particle has mass m. The potential is quadratic around its center, $V(x - x_c) = \frac{1}{2}k(x - x_c)^2$; however, the center moves at a constant velocity, $x_c(t) = vt$, as shown in the Figure.



(a) **[2 points]** Write down the time-dependent Schroedinger equation for the particle's wave function $\Psi(x, t)$ and its boundary conditions.

(b) **[3 points]** To solve the equation, use the ansatz $\Psi(x, t) = \Phi(x - vt)\mathrm{e}^{i(mvx-Et)/\hbar}$. Derive the equation for the function $\Phi(x)$ and its boundary conditions. Compare this equation to that for a particle in a static potential, and use the analogy to write down the energy levels (in terms of $\omega = \sqrt{k/m}$).

(c) **[3 points]** Suppose that the particle is in the ground state. Find the corresponding eigenfunction $\Phi_0(x)$ by solving the above equation. (Hint: the ground state wave function is Gaussian, so you may use the ansatz $\Phi_0(x) \propto \mathrm{e}^{-\frac{x^2}{2\sigma^2}}$ and find what σ^2 should be.) Use the result to write the full solution to the time-dependent Schroedinger equation.

(d) **[2 points]** Now suppose that at time $t = 0$ the potential well stops moving immediately and remains static afterwards. Find the probability that the particle will later be in the ground state of the static potential.