Student ID Number: __________

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part A, 8 January 2001, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to one of the examination monitors.
6. Each problem is worth 10 points.

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. Consider a three dimensional, nonrelativistic particle of mass m which moves in the central potential,

$$V(r) = -\frac{a}{r} + br \ ,$$

where a and b are positive constants. Note that, for real α, the equation $x^3 + x = \frac{2}{27}\alpha$ has the unique, real solution,

$$x(\alpha) = -\Big[\alpha + \sqrt{27+\alpha^2}\Big]^{-\frac{1}{3}} + \frac{1}{3}\Big[\alpha + \sqrt{27+\alpha^2}\Big]^{\frac{1}{3}} \ .$$

(a) (3 points) What are the allowed energies with Bohr's quantization condition ($mvr = \hbar n$)? Give the limiting forms and first order corrections for the three regimes defined by small a, by small b, and by large n.

(b) (4 points) Find the ground state energy and the normalized wave function for $b = 0$.

(c) (3 points) Compute the perturbative correction to the ground state energy to first order in b.

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A2. (10 points) You are given a battery E, a voltmeter V, an ammeter A, and two resistors R_1 and R_2. The battery, voltmeter, and ammeter are not ideal, and their internal resistances are known, approximately, to be $R_E \approx 1$ kΩ, $R_A \approx 1\ \Omega$, $R_V \approx 1$ MΩ. The values of the resistors are known only approximately as well, and they are $R_1 \approx 100\ \Omega$ and $R_2 \approx 100$ kΩ. For each of the resistors, construct a circuit (using E, V, A, and either R_1 and R_2), to measure their resistance with the least possible relative error. In each case, estimate the expected errors $\Delta R_1/R_1$ and $\Delta R_2/R_2$. Please note that only one measurement per resistor is allowed.

A3. The Fermi-Dirac distribution function is

$$\mathcal{F}_{F-D} = \frac{1}{\exp\left[\frac{E-\mu}{k_B T}\right] + 1} ,$$

where $\mu(T=0) = E_F$. Consider a metal whose density of states for electrons (for $E \geq 0$) is

$$g(E) = \frac{3N}{2} (E_F)^{-3/2} E^{1/2} ,$$

where N is the number of electrons in the system.

(a) (2 points) Plot the F-D function in the limit $T \to 0$. Show your derivation. Explain what E_F represents in this limit.

(b) (3 points) For the $T = 0$ case of the electrons in a metal as described above, calculate the mean energy per electron $\bar{E}$ in terms of E_F.

(c) (2 points) Find U, the internal energy of the system of electrons at $T = 0$.

(d) (3 points) In the low temperature limit, find the approximate, *i.e.* temperature dependent, electronic contribution to the metal's heat capacity due to the N electrons.