Student ID Number: ________

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part A, 6 January 2003, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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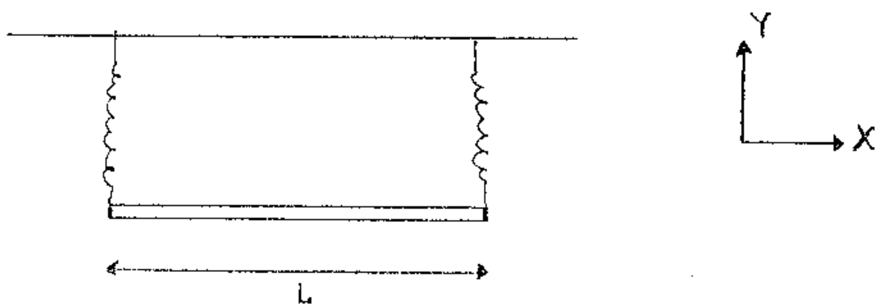
A1. A rigid uniform bar of mass M and length L is supported in equilibrium in a horizontal position by two massless springs attached one at each end, see Figure below. The springs have the same spring constant k and are kept vertical by attaching the other ends on a frictionless track. The center of the gravity is constrained to move parallel to the vertical Y axis. The motion of the bar is restricted to the XY-plane.

(a) (3 points) Write down the equations of motion of the system.

(b) (3 points) Find the normal modes of the system.

(c) (2 points) What are the frequencies of vibration of the system?

(d) (2 points) If one end is pushed down by a distance of h and then released, what is the subsequent motion of the system?



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A2. A coaxial cable can be modeled by a ladder network composed of series inductors and parallel capacitors, see Figure 1 below. Each section of an inductor and a capacitor represents a unit length of the cable where L and C are the inductance and capacitance *per unit length*, respectively. Then, the transmission line has the characteristic impedance, Z_0, and the phase velocity, ν, which can be written as

$$Z_0 = \sqrt{\frac{L}{C}} \quad \text{and} \quad \nu = \sqrt{\frac{1}{LC}} \quad .$$

(a) (3 points) RG-58 coaxial cable has a characteristic impedance of 50 Ω and a typical line capacitance of 93.5 pF/m. What is the phase velocity of the signal in this cable?

(b) (3 points) When one end of the coaxial cable with characteristic impedance Z_0 is terminated with a load impedance Z_ℓ, a portion of the RF signal injected into the other end is reflected at the terminal end due to the impedance mismatch, see Figure 2 below. Show that

$$\frac{A_r}{A_i} = \frac{Z_\ell - Z_0}{Z_\ell + Z_0} \quad ,$$

where A_r is the amplitude of the reflected signal and A_i is the amplitude of the incident signal.

(c) (4 points) Use the previous two results of this problem to answer this last question. One end of a 50 m long RG-58 cable is shorted as shown in Figure 3. When a step voltage of 1 V is applied to the other end of the cable through a 50 Ω resistor, a short pulse is detected at point X. Explain why a short pulse appears at X and estimate the height and length of the pulse.

Figure 1

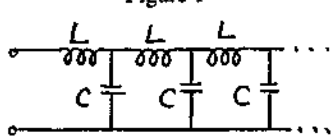


Figure 2

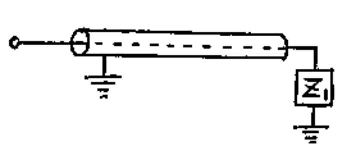
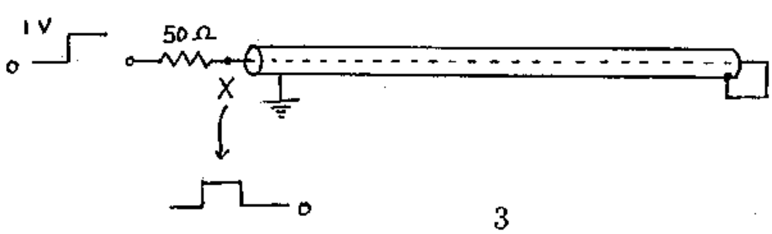


Figure 3



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A3. A living cell is surrounded by a membrane that is very thin ($d \simeq 6 - 8$ nm) relative to the diameter of the cell. Although the regions near the membrane are good conductors, the membrane itself is a poor conductor. A potential difference V usually exists across the membrane, with the outside of the cell typically around $\simeq 70$ mV more positive than the inside (see Figure 1 below).

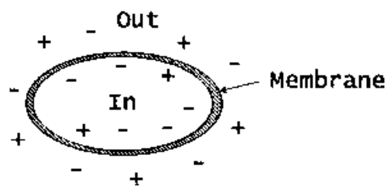

Figure 1: The living cell (simplified).

(a) (3 points) Consider the small ions (such as Na^+, Cl^-, K^+, etc.) in solution near the membrane. Inside the cell, each ion species i has an average concentration $C_i(0)$. At any finite temperature T, some of these ions will enter the membrane and diffuse a distance x up the potential gradient. For an ion species i that carries a charge $Z_i e$, give an expression for the average concentration $C_i(x)$ of that ion ($0 \leq x \leq d$) in terms of Z_i, $V(x)$, $C_i(0)$, and any relevant physical constants.

(b) (1 point) Use your expression to estimate the ratio $C_{Na}(d)/C_{Na}(0)$ for the sodium ion Na^+ in a typical cell at room temperature.

(c) (3 points) The membrane has a small finite conductivity σ_i for the ion species i, and the voltage V creates an ion electric current j_i across the membrane. However, at any point x within the membrane the concentration gradient drives an opposing ion current j_D due to Fick's Law of Diffusion:

$$j_D = -ZeD(\partial C/\partial x) \quad .$$

If the system has attained equilibrium and contains only one species of mobile ion, find the expression for the concentration gradient $\partial C/\partial x$ in terms of the potential gradient $-\partial V/\partial x$ within the membrane.

(d) (3 points) Now find the simple expression that shows that the conductivity σ_i associated with an ion within the membrane depends only on its local concentration $C_i(x)$, diffusion constant D, and physical constants.