Student ID Number: _____

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part A, 5 January 2004, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. Consider a nonrelativistic particle of mass m moving in one dimension in the potential

$$V(x) = \frac{g^2}{12}x^4 - \frac{f^3}{3g}x \quad .$$

(a) (4 points) Calculate the splitting between the lowest two energy levels of this system as $g \to 0$. (*Hint:* Expand the potential about its minimum.)

(b) (3 points) The largest non-vanishing correction to the splitting is of order g^p for $p > 0$. Find p, and justify your answer.

(c) (3 points) Without doing a detailed calculation of this correction to the splitting, completely determine its dependence on $g, f, m, \hbar$.

A2. (10 points) Consider the following one-dimensional model of a rubber band, as shown in Figure 1. Let n_+ be the number of links in the $+z$ direction and let n_- be the number of links in the $-z$ direction. All links have to be directed in either the positive or negative z direction. Each link has a length d. The total number of links is N. Starting from the change in the internal energy, namely

$$\mathrm{d}U = T\,\mathrm{d}S + \mathcal{F}\,\mathrm{d}\ell \quad ,$$

where ℓ is the total extension of the rubber band from one end to the other, $\mathcal{F}$ is the tension in the rubber band, and the rest of the notation is standard. Show that the equation for the force of tension $\mathcal{F}$ may be written as

$$\mathcal{F} = \frac{k_B T}{2d} \ln\left(\frac{Nd + \ell}{Nd - \ell}\right) \quad .$$

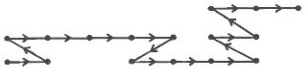

Figure 1. A collection of links representing a simplified model of a rubber band. The links lie in the $+z$ and $-z$ directions. Some of the links are drawn at an angle so the arrangement may be seen more clearly. Figure from R. Bowley and M. Sánchez, *Introductory Statistical Mechanics*, Oxford Science Publications.

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A3. (10 points) If you start a coin rolling on a table with care, you can make it roll in a circle with the axis of the coin leaning inward by an angle ϕ. The mass of the coin is m, the radius of the coin is b, the radius of the circle it follows on the table is R, and its speed is v.

(a) (3 points) Derive or write down the inertia tensor for the coin with respect to the principle axes.

(b) (3 points) In terms of the specified variables, give the condition when the coin will roll without slipping.

(c) (4 points) Find the angle ϕ that the axis makes with the horizontal when $b \ll R$.