Student ID Number: _________

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part C, 6 January 2004, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. Consider the three spin-1 matrices

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \; ; \; S_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \; ; \; S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \; ;$$

which represent the components of the spin angular momentum of some elemenary particle at rest along the x, y, and z axes, respectively.

(a) (6 points) What are the possible values we can get if we measure the spin along the x-axis?

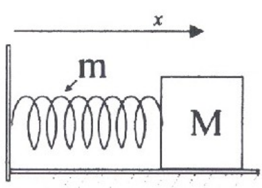
(b) (4 points) Suppose we obtain the largest possible value when we measure the spin along the x-axis. If we now measure the spin along the z-axis, what are the probabilities for the various outcomes?

C2. A uniform spring has a mass m, length ℓ, and a spring constant k. This spring has a block of mass M attached to it as shown in the figure. There is no friction between the mass M and the surface of the horizontal table. This system undergoes simple harmonic oscillations.

(a) (4 points) Find the maximum value of the potential energy of this system in terms of the spring constant k and the amplitude of oscillation A.

(b) (4 points) Find the maximum value of the kinetic energy in terms of m, M, and v_{max}, the maximum velocity of the mass M.

(c) (2 points) What is the frequency of oscillation of this sytem?



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C3. Consider two planar current loops, with currents I_1 and I_2, and origins O_1 and O_2, respectively. These loops are disconnected, but otherwise can have arbitrary shapes. $\mathbf{R}$ is the relative vector of these origins, directed from O_2 to O_1.

(a) (3 points) Now consider two current elements $I_1 d\mathbf{l}_1$ and $I_2 d\mathbf{l}_2$ on loops 1 and 2, respectively, with relative position vector

$$\mathbf{r}_{12} = \mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2 \quad ,$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of these current elements relative to O_1 and O_2, respectively. Using the Biot-Savart law and the Lorentz force, show that that the force $d\mathbf{F}_{12}$ on the first loop's current element due to the second loop's current element is

$$d\mathbf{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \frac{d\mathbf{l}_1 \times (d\mathbf{l}_2 \times \mathbf{r}_{12})}{r_{12}^3} \quad .$$

(b) (3 points) By performing the integrals over the loops 1 and 2, show that the total force between them is

$$\mathbf{F}_{12}(\mathbf{R}) = -\frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r_{12}^3} \, \mathbf{r}_{12} \quad .$$

Hints: Use

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

and

$$\frac{d\mathbf{l}_1 \cdot \mathbf{r}_{12}}{r_{12}^3} = -d(\frac{1}{r_{12}}) \quad ,$$

which is the total differential in the integral over $d\mathbf{l}_1$.

(c) (1 point) Using the mutual inductance, namely

$$M_{12}(\mathbf{R}) = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{|\mathbf{r}_1 - \mathbf{r}_2 + \mathbf{R}|} \quad ,$$

show that

$$F_{12}(\mathbf{R}) = I_1 I_2 \nabla_R M_{12}(\mathbf{R}) \quad .$$

(d) (2 points) Show that

$$\nabla_R^2 M_{12}(\mathbf{R}) = 0 \quad .$$

(e) (1 point) Assume that the current loops are in planes perpendicular to each other. For this case, show that the self-induction $M_{12}(\mathbf{R})$ vanishes.