

Student ID Number: _____

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

Part B, 4 January 2004, 14:00 - 17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

Part B, 4 January 2005, 14:00 - 17:00

B1. A particle of mass m moves in a harmonic oscillator potential

$$V(x) = \frac{1}{2} m \omega^2 x^2 .$$

Let \mathcal{H} be the oscillator Hamiltonian and u_n be the eigenfunction of the n -th oscillator, such that

$$\mathcal{H} u_n = E_n u_n , \quad \text{with } \langle u_n | u_m \rangle = \delta_{nm} \quad \text{and} \quad E_n = \hbar \omega \left(n + \frac{1}{2} \right) .$$

The raising and lowering operators \hat{a} and \hat{a}^\dagger are given by

$$\hat{a} u_n = \sqrt{n} u_{n-1} \quad \text{and} \quad \hat{a}^\dagger u_n = \sqrt{n+1} u_{n+1} , \quad \text{with } x = \frac{x_0}{\sqrt{2}} (\hat{a} + \hat{a}^\dagger) .$$

Suppose at time $t = 0$, the particle is in the state

$$\psi(x, 0) = \frac{1}{\sqrt{2}} [u_1(x) + u_2(x)] ,$$

where u_1 and u_2 are real.

(a) (2 points) Find the state of the particle $\psi(x, t)$ at a later time t .

(b) (2 points) Show that the expectation value $\langle x^r \rangle$ is given by

$$\langle x^r \rangle = \frac{1}{2} [\langle u_1 | x^r | u_1 \rangle + \langle u_2 | x^r | u_2 \rangle + 2 \cos \omega t \langle u_2 | x^r | u_1 \rangle] .$$

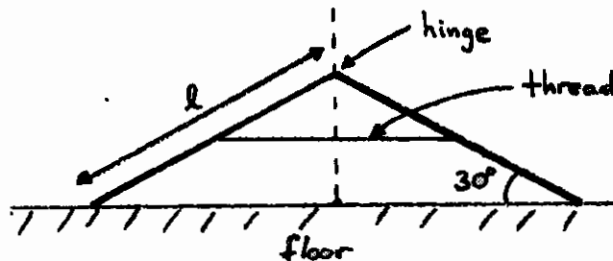
(c) (3 points) Obtain the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$ in terms of x_0 , ω , and t .

(d) (3 points) Now suppose ϕ is the normalized solution of $\hat{a}\phi = \lambda\phi$, where λ is a given real number. Find the probability, for general n , of obtaining the value E_n on measuring the energy in the state ϕ . [Hint: Expand $\phi = \sum b_n u_n$ and find the recursion relation satisfied by b_n . Use the recursion relation to write b_n in terms of b_0 . Use normalization conditions to find b_0 .]

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part B, 4 January 2005, 14:00 - 17:00

B2. Two identical, thin rods, each of mass m and length ℓ , are connected to an ideal (frictionless and massless) hinge and a horizontal thread. The system rests on a smooth surface as shown in the figure. At time $t = 0$, the thread is cut. Neglect the mass of the thread, and consider only motion in the $x - y$ plane.

- (a) (6 points) Find the speed at which the hinge hits the floor.
- (b) (4 points) Find the acceleration of the hinge just before it hits the floor.



- B3. (a) (2 points) What is the potential energy of a conducting sphere of radius r and charge q ? You might want to recall that the potential energy may be defined as the amount of work that needs to be done to charge the sphere by bringing small incremental charges from infinitely far away.
- (b) (4 points) Consider two conducting spheres of radii r_1 and r_2 . The distance between them is d with $d \gg r_1, r_2$. Calculate the amount of energy that needs to be done to place a charge q_1 on the sphere of radius r_1 and a charge q_2 on the sphere of radius r_2 . The charges are brought from infinity. Show that the answer does not depend on the order in which charges are placed on one sphere or the other.
- (c) (2 points) The two spheres in part (b) are now connected by a fine wire so the charge can flow from one to the other. What are the final values of q_1 and q_2 in terms of the total charge $Q = q_1 + q_2$?
- (d) (2 points) Consider a conducting sphere of radius r at a distance d from an infinite conducting plane, with $d \gg r$. How much work needs to be done to place charge q on the sphere now?