

Student ID Number: _____

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part C, 5 January 2005, 9:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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- C1. The Stark effect is the perturbation, due to an electric field, of the energies and wavefunctions of quantum mechanical systems. Consider the hydrogen atom in a constant electric field in the z-direction given by

$$\mathbf{E} = E_z \hat{\mathbf{z}} .$$

- (a) (2 points) If the electron mass is m , the nuclear mass is M , the elementary charge is e , write down the effective one-particle, unperturbed (or zeroth order) Schrödinger equation (i.e., no electric field is present yet) governing the internal structure of the hydrogen atom. You may use either MKS or cgs units, but you must specify your choice and be consistent.
- (b) (2 points) Now write down the perturbation Hamiltonian \mathcal{H}_1 due to the electric field \mathbf{E} .
- (c) (3 points) Write down expressions for the first-order energy of the ground state and for the first-order energies of the next four excited states. Note: the unperturbed first-excited state is four-fold degenerate, as spin degeneracy is not included.

The zeroth order ground state wavefunction and energy (in cgs units) are

$$\psi_{1s}^{(0)}(\mathbf{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \quad \text{and} \quad \epsilon_{1s}^{(0)} = -\frac{e^2}{2a_0} ,$$

where a_0 is the Bohr radius.

The zeroth order, first-excited state wavefunctions and energies (in cgs units) are

$$\psi_{2lm}^{(0)}(\mathbf{r}) = R_{2l}(r) Y_{lm}(\hat{\Omega}) \quad \text{and} \quad \epsilon_{2lm}^{(0)} = -\frac{e^2}{8a_0} ,$$

with

$$R_{20}(r) = \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0} ,$$

$$R_{21}(r) = \frac{1}{\sqrt{3}} \left(\frac{1}{2a_0}\right)^{3/2} \left(\frac{r}{a_0}\right) e^{-r/2a_0} ,$$

and $Y_{lm}(\hat{\Omega})$ are the spherical harmonics:

$$Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}} ,$$

$$Y_{1\pm 1}(\theta, \phi) = \mp \left(\frac{3}{8\pi}\right)^{1/2} \sin \theta e^{\pm i\phi} ,$$

and

$$Y_{10}(\theta, \phi) = \left(\frac{3}{4\pi}\right)^{1/2} \cos \theta .$$

- (d) (3 points) Now evaluate these expressions, and reduce them to formulas in terms of a_0 , e , and E_z . Hint: Use symmetry!!

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- C2. Consider two parallel plates placed 1 cm apart from each other. The space between the plates is filled by air at normal atmospheric pressure. The plates are kept at room temperature, but there is a small difference of 1°C between them. Using standard physical constants, estimate the rate of heat transfer [in units of $\text{J}/(\text{m}^2 \text{ s})$] between the plates produced by molecules of air colliding with the plates.

Hint: You should calculate the mean-free-path of the air molecules between collisions. Write down every step in your calculation. To avoid getting lost, it is advisable to keep track of the units in every equation.

You may (or may not) wish to recall:

Boltzmann's constant: $k_B = 1.3807 \times 10^{-23} \text{ J K}^{-1}$.

Avogadro's number: $N_A = 6.0221 \times 10^{23} \text{ particles mol}^{-1}$.

Average molecular weight of air: $MW_{\text{air}} \approx 30$.

Mass of the proton: $m_p = 1.6726 \times 10^{-27} \text{ kg}$.

You are expected to know (or easily calculate):

The approximate size of an atom.

A typical value of room temperature.

The approximate density of air at normal atmospheric pressure.

- C3. A spherical shell of radius R has a surface charge density given by

$$\sigma_\circ = \kappa \cos^2 \theta \quad ,$$

where κ is a constant.

- (2 points) What is the expression for $\sigma_\circ(\theta)$ in terms of Legendre polynomials?
- (4 points) Find the potential inside and outside the shell.
- (4 points) Find the electric field inside and outside the shell.