Student ID Number: ________

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part D, 5 January 2005, 14:00 - 17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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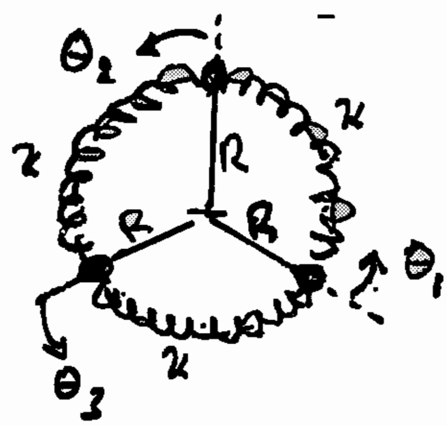
D1. A simplified, classical mechanical model of a homonuclear triatomic molecule consists of three equal point masses m which slide without friction on a fixed circular loop of radius R. The masses are connected with identical springs of spring constant κ. The springs also slide freely on the circular loop. The masses have angular positions θ_1, θ_2, and θ_3 measured from equally spaced rest positions. See the diagram. In this problem, you may ignore gravity.

(a) (2 points) Write the Lagrangian for this system.

(b) (2 points) From the Lagrangian, obtain the equations of motion for θ_i, $i = 1, 2, 3$.

(c) (3 points) Show that the mode of oscillation for which $\theta_1 = \theta_2 = \theta_3$ corresponds to constant total angular momentum.

(d) (3 points) For the situation in which the total angular momentum is zero and simultaneously $\theta_1 + \theta_2 + \theta_3 = 0$, show that there are two degenerate oscillatory modes and determine their common frequency.



PRELIMINARY EXAMINATION
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Part D, 5 August 2005, 14:00 - 17:00

D2. A model for ferromagnetism is given by the Hamiltonian

$$\mathcal{H} = -\mu_0 (H_0 + H_m) \sum_{i=1}^{N} \sigma_i,$$

where μ_0 is the Bohr magneton and σ_i is a spin variable (representing the magnetic moment of the atom i) which takes the values $\pm\frac{1}{2}$. In addition, H_0 is an applied external magnetic field, while H_m is the average "mean field" of the neighbors for each atom, such that

$$\mu_0 H_m = 2J \sum_{i=1}^{2} \langle \sigma_i \rangle \quad .$$

Here, $\langle \sigma_i \rangle$ is the equilibrium average of the spin, and J is the coupling constant among neighbors.

(a) (5 points) For the special case of $J = 0$ (non-interacting spins), calculate the partition function, the free energy, and the total magnetic moment

$$M = \mu_0 \sum_{i=1}^{N} \langle \sigma_i \rangle$$

for weak external fields ($\beta H_0 << 1$, $\beta = 1/k_B T$).

(b) (5 points) Repeat the calculation for $J \neq 0$ and show that $M \neq 0$ for $H_0 = 0$, as $T \to 0$ (ferromagnetism).

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

Part D, 5 January 2005, 14:00 - 17:00

D3. Consider a non-relativistic electron of mass m and charge $-e$ rotating in a circular orbit in a field of a heavy nucleus of mass $M >> m$ and charge Ze. Moving along the circular path, the electron experiences acceleration, emits electromagnetic radiation at the rotational frequency ω, and slowly (adiabatically) spirals inward.

(a) (3 points) Show that in addition to dissipating its energy, the system is losing its angular momentum.

(b) (5 points) Find the relationship between the rates of energy loss $-\mathrm{d}E/\mathrm{d}t$ and angular momentum loss $-\mathrm{d}L/\mathrm{d}t$.

(c) (2 points) What is the quantum mechanical relationship between the energy and spin of a photon?