Student ID Number: __________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, 5 January 2006, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, 5 January 2006, 14:00–17:00

B1. Consider a one-dimensional charged harmonic oscillator in a static electric field. The Hamiltonian is

$$H = H_1 + H_2,$$

with the first term being the harmonic oscillator

$$H_1 = \frac{p^2}{2m} + \frac{1}{2} m\omega_0 x^2$$

with mass m and spring constant $k = m\omega_0^2$. The second term in H is the electrostatic potential

$$H_2 = -Q\mathcal{E}x$$

with Q the charge of the oscillating mass and $\mathcal{E}$ the electric field.

(a) (2 points) Solve the Hamiltonian H to find the exact ground-state energy E_g and the ground-state wave function $\psi_g(x)$. (You may use an un-normalized wave function for this part.)

Hint: Define $z = x + C\mathcal{E}$, where C is some constant. Write H in terms of z and adjust C until H reduces to a form you know how to solve.

(b) (2 points) Calculate for the ground state the dipole moment $d = \langle \psi_g | ex | \psi_g \rangle$ using your solutions to part (a).

(c) (2 points) Now, treating H_2 as a perturbation to H_1, use perturbation theory to obtain the first non-zero correction to the ground-state energy and compare your result to what you found in part (a).

(d) (2 points) Continuing with the perturbation approach, find the first correction, $\delta\psi$, to the ground-state wave function ψ_0.

(e) (2 points) Using your result for $\psi_0 + \delta\psi$ compute the dipole moment d to first order in $\mathcal{E}$ and compare your result to what you found in part (b).

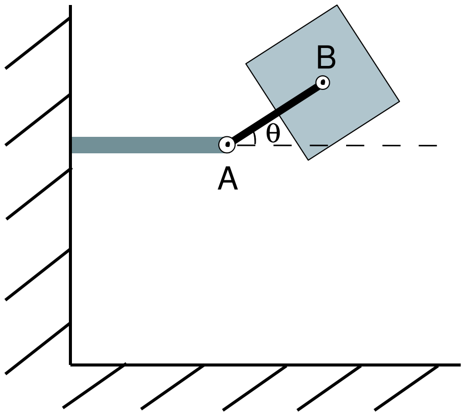
Note: The standard solution to the simple harmonic oscillator, Hamiltonian H_1, finds the ground-state wave function to be

$$\psi_0(x) = A_0 e^{-\frac{m\omega_0}{2\hbar}x^2} \quad \text{where} \quad A_0 = \left(\frac{m\omega_0}{\pi\hbar}\right)^{\frac{1}{4}},$$

with energy $E_0 = \hbar\omega_0/2$. You will find that parts (c)–(e) of the problem are considerably simplified if you use raising and lowering operators as much as possible. These are:

$$a_{\pm} = \sqrt{\frac{m\omega_0}{2\hbar}}x \mp i\frac{p}{\sqrt{2m\omega_0\hbar}}.$$

B2. In the apparatus shown, a thin rod of mass M and length L is attached at point A to a hinge around which it rotates freely. At the other end of the rod (point B) a thin, square-shaped body of mass M and sides L is attached tightly so that it cannot rotate relative to the rod. The hinge connects the rod to a rigid arm that is attached to a supporting wall. The arm is long enough so that the rod and the mass can swing freely without hitting the wall.



(a) (3 points) If the rod is hanging from the support and given a slight nudge, find the period of small oscillations.

(b) (4 points) The rod is raised to the position shown, at an angle θ above the horizontal, and released. Find the direction and magnitude of the force that the hinge exerts on the rod when the rod swings to the horizontal position. Show the direction clearly on the diagram.

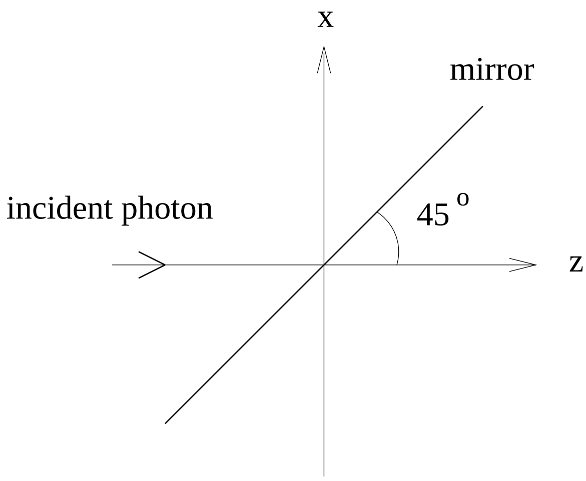
(c) (3 points) Redo (a) assuming now that at point B the square mass is attached to the rod by a frictionless bearing so that it can rotate freely.

Note: $I_{rod} = ML^2/12$ for a rod of length L (about an axis through the center of the rod perpendicular to the rod); $I_{square} = Ma^2/6$ for a square of sides a (about an axis through the center of the square perpendicular to the plane).

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, 5 January 2006, 14:00–17:00

B3. A mirror, which is perpendicular to the x-z plane, makes an angle of 45° with the z axis in the mirror's frame as shown in the figure. The motion of the mirror is in the z direction with a velocity v in the lab frame. A photon of energy e_i moving in the positive z direction is reflected by the mirror.



(a) (3 points) What is the energy e'_f of the reflected photon in the mirror's frame?

(b) (2 points) What is the energy e_f of the reflected photon in the lab frame?

(c) (3 points) What angle does the momentum of the reflected photon make with the x axis in the lab frame?

(d) (2 points) What angle does the mirror make with the z axis in the lab frame?