Student ID Number: _____

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part C, 6 January 2006, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. Consider a Hilbert space spanned by two independent harmonic oscillators. Its ground state satisfies

$$a\,|\,0\rangle \;=\; b\,|\,0\rangle \;=\; 0\,, \qquad \langle 0\,|\,0\rangle \;=\; 1\,,$$

where

$$[\,a\,,\,a^\dagger\,] \;=\; [\,b\,,\,b^\dagger\,] \;=\; 1\,,$$

all other commutators being zero. Consider the new operator

$$A \;=\; \alpha\,a^\dagger \;+\; \beta\,b\,,$$

where α and β are real, and its hermitian conjugate $A^\dagger$.

(a) (2 points) Find the constraints α and β must satisfy for A and $A^\dagger$ to be a canonical pair of ladder operators.

(b) (2 points) Find the expression for the second canonical set, B and $B^\dagger$, which commute with A and $A^\dagger$.

(c) (2 points) Express the new vacuum state $|\,\Omega\rangle$, which satisfies

$$A\,|\,\Omega\rangle \;=\; B\,|\,\Omega\rangle \;=\; 0\,,$$

in terms of the original oscillator states.

(d) (4 points) Evaluate the value of the number operator $a^\dagger\,a$ in the state $|\Omega\rangle$. What is the significance of this state?

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C2. A paramagnetic salt contains N atoms, each of which has an intrinsic magnetic moment $\boldsymbol{\mu}_i$ ($i = 1, 2, \ldots, N$). In the presence of an external magnetic field $\mathbf{H}$ the Hamiltonian is

$$\mathcal{H} = -\sum_{i=1}^{N} \boldsymbol{\mu}_i \cdot \mathbf{H} = -\mu H \sum_{i=1}^{N} \cos\theta_i,$$

with θ_i the angle between the moment of the ith spin and the external field. Assume that these angles can be treated as classical coordinates.

(a) (3 points) Calculate the canonical partition function and the free energy for the spins.

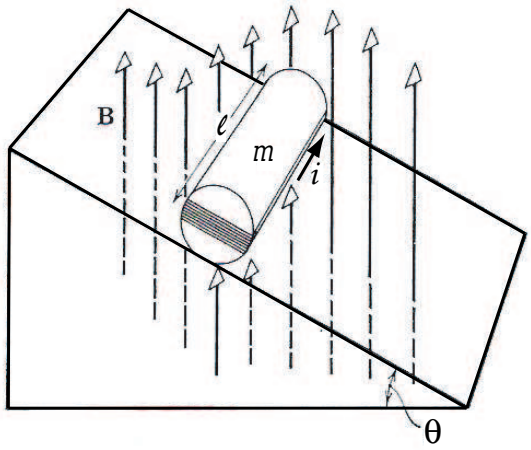
(b) (3 points) Calculate the induced magnetic moment (per spin) along the direction of the external field:

$$m = \frac{1}{N}\langle \sum_{i=1}^{N} \mu \cos\theta_i \rangle.$$

(c) (2 points) Calculate $\chi = \partial m/\partial H$, the spin susceptibility per atom.

(d) (2 points) Find the limiting (i) low-temperature and (ii) high-temperature behaviors of the susceptibility, and (iii) sketch the temperature dependence of χ using these results.

C3. (a) (6 points) Please refer to the sketch below.



A wooden cylinder of mass m, length l and radius R lies on a plane inclined at angle θ with the horizontal, as shown. The cylinder is wrapped with a conducting coil of N turns; the plane of the coil coincides with the cylinder axis and is parallel to the incline. The entire system is immersed in a uniform vertical magnetic field B, as shown.

Calculate the minimum current i through the coil in the direction shown, such that the cylinder does not roll down the plane.

(b) (4 points) A sealed box with two external terminals contains an unknown combination of emfs and resistances. If (i) a 10-Ω resistance connected across the terminals draws a current of 1 A, and (ii) an 18-Ω resistance draws a current of 0.6 A, what resistance will draw 0.1 A of current?