Student ID Number: ________

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part A, January 4, 2023, 9:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.”

DO NOT OPEN EXAM UNTIL INSTRUCTED
A1. The wave function of a particle is $\psi(x) = Ae^{-ax}$ for $x > 0$ and $\psi(x) = Ae^{ax}$ for $x < 0$ with $a > 0$.

(a) [3 points] For $x > 0$, use this wave function in the time-independent Schrödinger equation with unknown potential $U(x)$. Simplify it to derive $U(x)$ for $x > 0$ when the energy $E$ is known.

(b) [3 points] For $x < 0$, use this wave function in the time-independent Schrödinger equation with unknown potential $U(x)$. Simplify it to derive $U(x)$ for $x < 0$ when the energy $E$ is known.

(c) [2 points] Find the normalization constant $A$.

(d) [2 points] Find the corresponding potential energy $U(x)$ at $x = 0$. 
A2. Consider a hydrogen atom immersed in a weak uniform electric field \( \mathbf{E} = \mathcal{E} \hat{z} \). Treat the hydrogen atom as a proton and electron bound by the Coulomb potential, neglecting all relativistic and spin orbit corrections, and you may regard the electron spin fixed \( m_s = +1/2 \) throughout this problem.

(a) [3 points] First recall the spectrum at zero field. Ignoring spin, give the degeneracies of the \( n = 1, 2 \) levels, and the angular momentum quantum numbers \( l, m_l \) and parity \( \pm \) of every state belonging to each of these energy levels.

(b) [3 points] Explain, without detailed calculation, why the energy shift of the \( n = 1 \) level due to the electric field is quadratic in the field \( \Delta E_{n=1} = -k \mathcal{E}^2 \) for very weak field, and why \( k \) is positive.

(c) [2 points] Now consider the \( n = 2 \) energy level in light of its degeneracies. Explain why the weak electric field will split some of the degeneracy by an amount linear in the electric field.

(d) [2 points] Calculate the first order splittings of the \( n = 2 \) levels of hydrogen in a weak electric field. The zeroth order wave functions you might need are:

\[
\psi_{210} = \sqrt{\frac{1}{32\pi a^5}} r \cos \theta e^{-r/2a}, \quad \psi_{200} = \sqrt{\frac{1}{32\pi a^5}} (2a - r) e^{-r/2a}
\]

where \( a \) is the Bohr radius.
A3. Consider a spin-1/2 particle moving in one spatial dimension with Hamiltonian given by

\[ \hat{H} = \hat{p}\hat{\sigma}_x + m\hat{\sigma}_z, \]

where \( \hat{p} = -i\partial_x \) is the momentum operator, \( m \) is the mass of the particle, and \( \hat{\sigma}_{x,z} \) are the two Pauli matrices.

This Hamiltonian describes a relativistic fermion in a one-dimensional universe, but you will not need any knowledge of relativistic quantum mechanics. All you need is the quantum mechanics of spin and the harmonic oscillator!

(a) [3 points] In usual cases, the mass \( m \) is independent of the position \( x \). Prove that \( \hat{H}^2 = \hat{p}^2 + m^2 \). Use this expression to find the two eigenvalues of \( \hat{H} \) for momentum eigenstates with eigenvalue \( p_0 \).

(b) [4 points] In certain cases, the mass \( m \) may depend on position, in which case \( \hat{p} \) no longer commutes with \( m \). Specifically, let \( m(x) = x \). Expand the operator \( \hat{H}^2 \) as a sum of three terms. (You will need \( [x,\hat{p}] = i \) and \( \hat{\sigma}_z \hat{\sigma}_x = -\hat{\sigma}_x \hat{\sigma}_z = i\hat{\sigma}_y \).)

(c) [3 points] By analogy with harmonic oscillators, find all eigenvalues of \( \hat{H}^2 \). Show that the smallest absolute value of the eigenvalues of \( \hat{H} \) for \( m(x) = x \) is zero.