Student ID Number: ________

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part B, January 3, 2024, 14:00–17:00

Instructions

1. You may use a non-programmable calculator (i.e. one that cannot store formulas). No other tables or aids are allowed or required.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.”

DO NOT OPEN EXAM UNTIL INSTRUCTED
B1. A wire with cross sectional area \( A \), conductivity \( \sigma \), and mass density \( \rho \) is formed into a vertical square loop of side length \( l \) and partially embedded in a region of uniform horizontal magnetic field \( B \), as shown in the figure below. Outside the region of uniform field, shown as the shaded region in the figure, the magnetic field is zero. At time \( t = 0 \) the loop is released. Gravity causes the loop to accelerate downwards.

(a) [3 points] Calculate the magnitude and direction of the current flowing in the wire when it is moving at speed \( v \) downwards.

(b) [3 points] What is the magnitude and direction of the magnetic force acting on the wire at this instant?

(c) [2 points] Write down the equation of motion of the loop.

(d) [2 points] Find the terminal velocity \( v_T \) of the loop in terms of the variables already listed and the acceleration due to gravity \( g \).
B2. (a) [3 points] The capacitance is defined as the proportionality constant between the electrostatic potential difference between two conductors and the accumulated charge: \( \Delta V = \frac{Q}{C} \). With that in mind, derive the capacitance of a parallel plate capacitor of area \( A \) and separation \( d \). You can assume that the two plates are separated by an air gap and that \( d \) is small, such that edge effects can be neglected.

(b) [2 points] Derive the equation for the addition of capacitors in parallel. This can be done either using geometric or circuit arguments.

(c) [5 points] A capacitor has square plates, each of side \( a \), making an angle \( \theta \) with one another, as shown in the figure. If the minimal separation between the plates is \( d \), show that for small angle \( \theta \), the capacitance is given by

\[
C = \frac{\epsilon_0 a^2}{d} \left( 1 - \frac{a\theta}{2d} \right).
\]

Hint: The capacitor may be divided into differential strips that are effectively in parallel.

[Diagram of the capacitor with side \( a \) and angle \( \theta \).]
B3. Suppose you take a plastic ring of radius $a$ and glue charge on it, so that the linear charge density is $\lambda_0 |\sin(\phi/2)|$. Then you spin the loop about its axis at an angular velocity of $\omega$.

(a) \textbf{[3 points]} What are the (time dependent) charge density $\rho(s, \phi, z, t)$ and current density $\vec{J}(s, \phi, z, t)$? Here $(s, \phi, z)$ are cylindrical coordinates, where the $z$ axis is the axis of the loop.

(b) \textbf{[3 points]} What is the retarded time $t_{\text{ret}}(\vec{r}, t; \vec{r}')$ from any point $\vec{r}' = a\hat{s}$ on the ring to the center of the ring $\vec{r} = 0$?

(c) \textbf{[2 points]} What is the retarded scalar potential (in Lorenz gauge) at the center of the ring?

(d) \textbf{[2 points]} What is the retarded vector potential (in Lorenz gauge) at the center of the ring?

Note 1: Unit vector $\hat{\phi} = -\sin(\phi)\hat{x} + \cos(\phi)\hat{y}$

Note 2: $\sin(A+B) = \sin A \cos B + \cos A \sin B$; $\cos(A+B) = \cos A \cos B - \sin A \sin B$