Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, January, 2012, 09:00–12:00

Instructions

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DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part A, January, 2012, 09:00–12:00

A1. A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane. How long Δt will it take for the charge to hit the plane, neglecting the force of gravity?

(a) **(3 points)** Dimensional analysis gives you how the time Δt depends upon the parameters q, m, d and ϵ_0, up to an overall constant. What is this form?

(b) **(3 points)** Suppose the charge q is at height z. Use the Method of Images to find the force exerted upon it by the conducting plane.

(c) **(4 points)** What is the exact formula for Δt?

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part A, January, 2012, 09:00–12:00

A2. Consider one dimensional infinite square well of width $2L$ with a particle of mass m moving in it ($-L < x < L$). The particle is in the lowest-energy state. Assume now that at $t = 0$ the walls of the well move instantaneously so that its width doubles ($-2L < x < 2L$). This change does not affect the state of the particle, which is the same before and immediately after the change.

(a) Find the wave function for the lowest-energy state and its energy at time $t < 0$.

(b) Find the eigenvalues and eigenfunctions of the modified system at time $t > 0$.

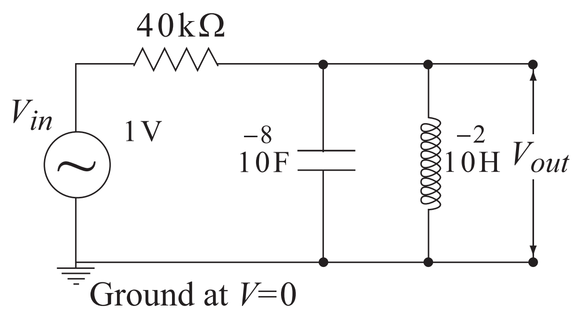
(c) Write down the wave function of the particle at times $t > 0$.

(d) Calculate the probability of finding the particle in an arbitrary eigenstate of the modified system.

(e) What is the probability of finding the particle in odd eigenstate?

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part A, January, 2012, 09:00–12:00

A3. A band pass filter that passes voltage signals over a narrow frequency range can be made from the combination of passive circuit elements (R, L, C) arranged as shown in the figure. The ac input voltage source V_{in} has a negligible output impedance and produces a sinusoidal signal $V_0 \sin(\omega t)$ with amplitude of $V_0 = 1$ volt that can be tuned over a wide range of frequencies $f = \omega/2\pi$. The output voltage detector V_{out} has high input impedance and can measure the amplitude $|V_{out}(\omega)|$ and phase $\phi(\omega)$ of the voltage at the dividing point between the resistance R and the complex impedance $Z_{LC}(\omega)$ of the parallel combination of the capacitance C and inductance L.



(a) **(2 points)** Calculate an expression for $Z_{LC}(\omega)$ and find the resonant frequency f_R in Hz where $Z_{LC}(\omega)$ is maximum. In the following sections leave all your answers in terms of the variables R, L and C.

(b) **(2 points)** Calculate the complex response $\dfrac{V_{out}(\omega)}{V_{in}(\omega)} = \left|\dfrac{V_{out}(\omega)}{V_{in}(\omega)}\right| \exp(i\phi(\omega))$ by separately calculating $\left|\dfrac{V_{out}(\omega)}{V_{in}(\omega)}\right|$ and $\phi(\omega)$ and then identify the resonant frequency where $\left|\dfrac{V_{out}(\omega)}{V_{in}(\omega)}\right|$ is maximum.

(c) **(2 points)** Sketch $\left|\dfrac{V_{out}(\omega)}{V_{in}(\omega)}\right|$ versus ω. In your sketch label all axes and locate the point where the maximum response occurs. Find the respective values of ϕ in the low and high frequency limits and at resonance.

(d) **(2 points)** By flipping a switch on the voltage source, a square wave with Fourier transform $V(t) = \dfrac{4}{\pi} V_0 \left[\sin(\omega t) + \dfrac{1}{3}\sin(3\omega t) + \dfrac{1}{5}\sin(5\omega t) + ...\right]$ is generated. Briefly describe how you (the experimentalist) can vary ω to check the values (1/3, 1/5, etc.) of the harmonic coefficients. You will want to use frequencies that give you maximum sensitivity as determined above.

(e) **(1 point)** Describe how you might obtain an experimental value for π. Remember that your voltage source can output both sinusoidal and square wave signals.

(f) **(1 point)** Rearrange the circuit elements in the figure and draw a sketch of a band reject filter, *i.e.*, a filter that rejects rather than accepts input voltages over a narrow frequency range.

Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, January, 2012, 14:00–17:00

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DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, January, 2012, 14:00–17:00

B1. A paramagnetic solid consists of N non-interacting spins fixed at different sites of a lattice. The z component of spin i can take one of three values: $S_i^z = 0$ or ± 1, where $i = 1 \to N$. In an external magnetic field B, this system is described by the Hamiltonian

$$H = -\gamma B \sum_{i=1}^{N} S_i^z,$$

where γ is a positive constant.

(a) **(2 points)** Calculate the canonical partition function at temperature T for the case of a single spin ($N = 1$).

(b) **(2 points)** Calculate the canonical partition function at temperature T for a general value of N.

(c) **(2 points)** Calculate the free-energy $F(T, B, N)$ of this system.

(d) **(2 points)** Calculate the magnetization $M(T, B, N)$ of this system.

(e) **(2 points)** Calculate the zero-field magnetic susceptibility $\chi(T, N)$ of this system.

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, January, 2012, 14:00–17:00

B2. (a) **(3 points)** A metal block with heat capacity C is initially at temperature T_1. It is placed in contact with a heat reservoir held at temperature T_2. The block comes to the temperature of the reservoir through quasi-static heat flow without the performance of any work. What is the total entropy change of the universe, as a result of this process? Note that T_1 may be greater or less than T_2.

(b) **(2 points)** Define $x = T_1/T_2$. Are there any values of x for which the total entropy change is negative? Prove it.

(c) **(5 points)** Now imagine that the same metal block (initially at temperature T_1) is connected to a reservoir (at temperature T_2 where $T_2 < T_1$) by means of an ideal reversible heat engine (e.g. Carnot engine). The engine runs and produces work until the block attains the temperature of the reservoir. How much work W is produced?

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part B, January, 2012, 14:00–17:00

Lorentz Transformation Of Electromagnetic Fields

B3. Consider a cylindrical charge distribution. Let the charge density drop as a Gaussian distribution of the distance from the z axis in the rest frame of the charge distribution (S):

$$\rho(\vec{r}) = Ae^{-(x^2+y^2)}$$

The distribution is passing by an observer, whose reference frame is S'. In the rest frame of the observer, the charge distribution is moving at a velocity $\vec{v} = v_z\hat{z}$.

(a) **(3 points)** Compute the electric fields $\vec{E}$ and $\vec{B}$ in the rest frame of the charge distribution, S.

(b) **(4 points)** Compute the charge and current densities ρ' and $\vec{j}'$ in the frame of the observer, S'.

(c) **(3 points)** Compute the electric ($\vec{E}'$) and magnetic ($\vec{B}'$) fields in the frame S'.

Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, January, 2012, 09:00–12:00

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DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, January, 2012, 09:00–12:00

C1. A charged pion (π^+) is a particle that has a mass of 139 MeV/c^2 and decays into a muon (mass 106 MeV/c^2) and a (mass-less) neutrino.

(a) **(3 points)** Calculate the momentum (in MeV/c) and energy (in MeV/c^2) of the muon and the neutrino in the rest frame of the pion.

(b) **(4 points)** The half-life of the π^+ is 2.5×10^{-8}s. In a particular experiment, half a stream of mono-energetic pions decay before they cover a distance of 15 meters. Calculate the speed of the pions (in terms of the speed of light, $c = 3 \times 10^8$ m/s.)

(c) **(3 points)** Neutral pions (π^0)s have a mass of 135 MeV/c^2 and decay very quickly into two γ rays. When an energetic π^0 is produced, the energy of the decay products, in the lab frame, depends on the angle between the direction of the decay products in the center-of-mass frame, and the direction of the center-of-mass motion in the lab frame If the π^0 has $\beta = 0.5$. Using the Lorentz transform, $p = \gamma(p' + vE'/c^2)$, or otherwise, what are the maximum and minimum energies of the decay products in MeV/c^2? ($\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$)

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

Part C, January, 2012, 09:00–12:00

C2. **(10 points)** What is the external magnetic field produced by a uniformly magnetized cylinder when $\vec{M}_0 = M_0\hat{x}$. Assume the cylinder extends infinitely along the z-axis and has a radius R. You may use any technique to generate the solution, but you must show all steps.

You might want to recall that the magnetic scalar potential, Φ_m, in cylindrical coordinates may be written as

$$\Phi_m(s,\phi) \;=\; a_o \;+\; b_o \ln s \;+\; \sum_{k=1}^{\infty} (a_k\, s^k \;+\; b_k\, s^{-k})\,(c_k\, \cos k\phi \;+\; d_k\, \sin k\phi) \qquad .$$

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, January, 2012, 09:00–12:00

C3. Most of the stars in a spiral galaxy, such as our Milky Way, are located in a spherical bulge at the galaxy center. A small number of stars are in the spiral arms extending out from the bulge. (Our sun is located in one of these spiral arms.) See Fig. 1 for a sketch of the galaxy as seen from far away.

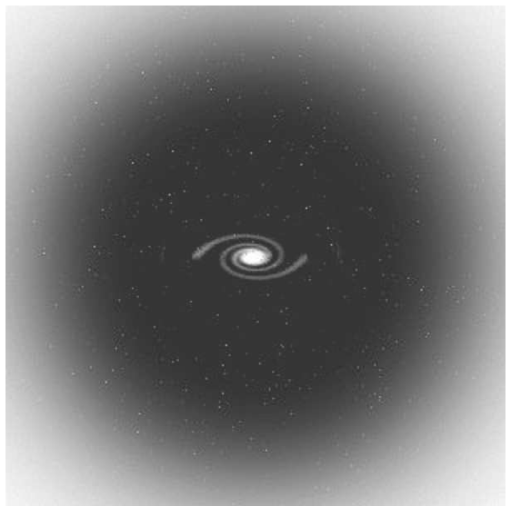

Fig. 1. A galaxy, viewed at a small angle away from directly above. The bulge is the bright sphere in the center, and two spiral arms extend out from the bulge. There is also a spherical "halo" or cloud of dark matter extending out from the bulge to a distance of $20\times$ the radius of the bulge, well beyond the ends of the spiral arms.

Let us make a model of the orbital motion of these stars. Make five simplifying assumptions: (1) The bulge is a sphere. (2) The number of stars in the bulge is so much larger than the number in the arms that the stars in the bulge control the orbital dynamics. (3) The stars in the bulge are uniformly distributed, so that the bulge has density

$$\rho = \frac{3M}{4\pi R^3}$$

where M is the total mass of the galaxy and R the radius of the bulge. (4) All stars are in circular orbits about the common center of mass. (5) Ignore for the moment the halo shown in Fig. 1.

(a) **(2 points)** Calculate the velocity of the stars inside the bulge as a function of their radius from the center, $r \le R$.

(b) **(2 points)** Calculate the velocity of the stars outside the bulge as a function of their radius from the center, $r \ge R$.

(c) **(2 points)** Make a sketch of the orbital velocities found in parts (a) and (b) as a function of orbit radius r. Use a scale of $0 \le r \le 9R$. What is the maximum value of the velocity, v_{max}? What is the value when $r = 9R$? What is the value when $r = 0$?

The actual plot of v versus r looks quite different than what you just plotted. A curve for one galaxy is shown in Fig. 2. The curve is quite flat for radii above 6 kpc, which we can take as the radius R of the bulge.

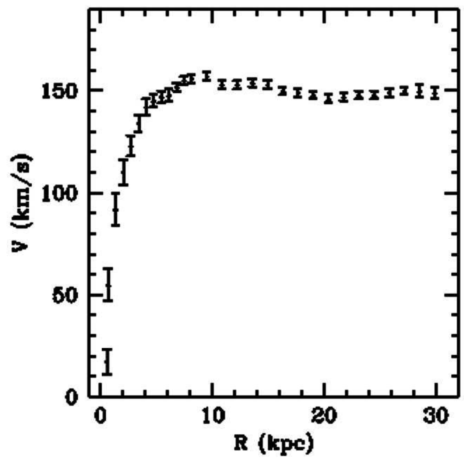

Fig. 2. Typical rotation curve (v versus r) of a galaxy.

We need to include the halo in our model in order to account for the flat portion of Fig. 2. Replace assumption (5) above with the following: (5) The spherical halo extends from the edge of the bulge, $r = R$, to $r = 20R$. (6) The halo density is *not* constant in r although it is constant as a function of angle at a given r. (7) The velocity is constant at v_{max} over $R \le r \le 20R$, where v_{max} is the velocity found in part (c) above.

(d) **(4 points)** Calculate the halo mass density outside the bulge as a function of distance r from the center. Here, $R \le r \le 20R$.

Student ID Number: ________

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part D, January, 2012, 014:00–17:00

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DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part D, January, 2012, 014:00–17:00

D1. A particle with energy E and mass m is in the one dimensional potential

$$\begin{aligned} V(x) &= \infty \text{ for } x < 0 \\ V(x) &= -V_0 \text{ for } 0 < x < a \\ V(x) &= 0 \text{ for } a < x. \end{aligned}$$

with $V_0 > 0$. In this problem we will find the bound states, $-V_0 < E < 0$.

(a) **(2 points)** Solve the one dimensional Schrodinger equation for ψ in the region $0 < x < a$. Given that $V(x) = \infty$ for $x < 0$, what is the general form of the solution in this region?

(b) **(2 points)** Solve the one dimensional Schrodinger equation in the region $x > a$. Given that we want the wave function to be normalizable, what is the general form of the solution in this region?

(c) **(2 points)** Match the boundary conditions at $r = a$ and derive a equation for the bound states.

(d) **(2 points)** Derive the condition on V_0 for there will not be a bound state for $-V_0 < E < 0$.

(e) **(2 points)** Derive the condition on V_0 for there to be only one bound state for $-V_0 < E < 0$.

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part D, January, 2012, 014:00–17:00

D2. Consider a harmonic oscillator which consists of a mass m and a spring with a spring constant k. The motion of the mass is restricted in the vertical direction (y-axis) by a frictionless guide going through the oscillator (see the figure below). A very light string of linear mass density ρ is attached directly to the mass and extends infinitely along the x-axis under constant tension T. If the spring is compressed from its initial equilibrium length and released, the mass oscillates and waves will propagate outward from the mass. Let $y(x,t)$ and $Y(t)$ denote the height of the string and the mass above the x-axis at time t, respectively. The connection point to the mass is specified as $x = 0$. Ignore the effect of the string *weight* (gravitational effect due to the string mass) in answering the following questions.

(a) Show that for small amplitude oscillations the equation of motion for the mass is given by

$$m\frac{d^2Y}{dt^2} + kY = T\left.\frac{\partial y}{\partial x}\right|_{x=0} .$$

(b) The waves propagating along the x-axis is governed by the wave equation

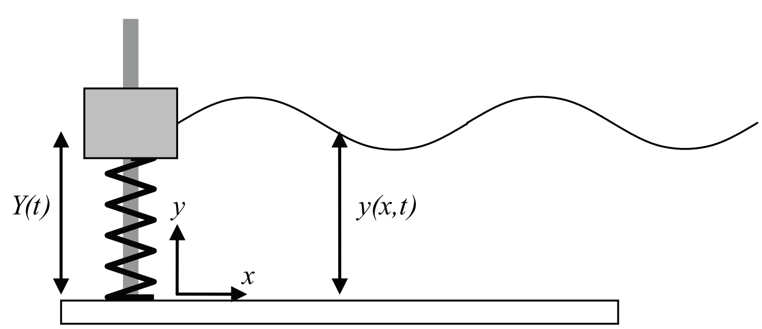
$$\left(\rho\frac{\partial^2}{\partial t^2} - T\frac{\partial^2}{\partial x^2}\right) y(x,t) = 0$$

Since the motion of the mass generates outgoing waves, the oscillator will experience damping (radiation damping), and the equation of motion will turn into the form of a damped harmonic oscillator:

$$m\frac{d^2Y}{dt^2} + \lambda\frac{dY}{dt} + kY = 0 .$$

Express λ in terms of the quantities given in the problem. *Do not derive the wave equation.*

(c) What is $E(t)/E(0)$ for weak damping, $\lambda^2 \ll km$? $E(t)$ is the total mechanical energy of the oscillator at time t.



PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part D, January, 2012, 014:00–17:00

D3. An electron is at rest in an oscillating magnetic field along the z-direction:

$$\vec{b} = B_0 \cos(\omega t)\hat{z}$$

where B_0 and ω are constants.

(a) **(2 points)** Construct the Hamiltonian matrix for this system.

(b) **(4 points)** If at $t = 0$ the electron starts out in the spin-up state with respect to the x-axis (that is: $\chi(0) = \chi_+^{(x)}$), determine $\chi(t)$ at any subsequent time.

Beware: This is a *time-dependent* Hamiltonian, so you cannot get $\chi(t)$ in the usual way from stationary states. However, in this case you can solve the *time-dependent* Schröedinger equation directly.

(c) **(2 points)** Show that if you measure S_x, the probability of getting $-\hbar/2$ is: $\sin^2\left(\frac{\lambda B_0}{2\omega}\sin(\omega t)\right)$.

(d) **(2 points)** What is the minimum field (B_0) required to force a complete flip in S_x?

Hint: you may find the Pauli spin matrices useful. $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and remember that $S = \left(\frac{\hbar}{2}\right)\sigma$.