Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, January 6, 2015, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. **QM - Sikivie**

(a) [**3 points**] A particle of mass m moving in one dimension, in a potential $V(x)$, is in a state of wavefunction $\Psi(x,t)$. Using the time-dependent Schrödinger equation, show that

$$\frac{\partial}{\partial t}|\Psi(x,t)|^2 = -\frac{\partial}{\partial x}j(x,t) \quad ,$$

where

$$j(x,t) = \frac{\hbar}{2mi}(\Psi^* \frac{\partial \Psi}{\partial x} - \Psi \frac{\partial \Psi^*}{\partial x}) \quad .$$

What is the physical meaning of $j(x,t)$?

(b) [**3 points**] In a region where $V(x)$ is constant (i.e. x-independent), the wavefunction has the form

$$\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-\frac{i}{\hbar}Et}$$

where A, B, k and E are constants. What is $j(x,t)$ in such a region?

(c) [**4 points**] The particle is incident from the left in a potential with a sudden drop:

$$\begin{aligned} V(x) &= 0 \qquad \text{for } x < 0 \\ &= -V_0 \qquad \text{for } x < 0 \end{aligned}$$

where $V_0 > 0$. As a function of the particle's energy, what is the probability that it is reflected? What is the probability that it is transmitted?

A2. **QM-Muttalib** Consider a quantum system described by the Hamiltonian

$$H = V_0 \begin{pmatrix} (1-\epsilon) & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}$$

where V_0 is a constant and ϵ is a small number. We will consider the limit $\epsilon = 0$ as the unperturbed Hamiltonian H_0, and compare the exact energy levels of H with those obtained from perturbation theory assuming $\epsilon \ll 1$.

(a) **(1 point)** Write down the eigenvalues and eigenvectors of the *unperturbed* Hamiltonian H_0.

(b) **(3 points)** Solve for the *exact* eigenvalues of H (for arbitrary ϵ).

(c) **(3 points)** Assuming $\epsilon \ll 1$, use first and second order *non-degenerate* perturbation theory to find the approximate eigenvalue for the state corresponding to the non-degenerate eigenvector of H_0. Compare your result with the corresponding exact solution obtained in (b).

(d) **(3 points)** Use *degenerate* perturbation theory to find the first order correction to the two initially degenerate eigenvalues. Compare your results with the exact solutions obtained in (b).

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A3. **Prelim Quantum Mechanics Question From Kevin**

An electron precessing in a time-independent magnetic field is described in the conventional eigenbasis of the $\hat{S}_z$ operator by a spinor

$$\chi(t) = \begin{pmatrix} \cos\omega t + \sin\omega t \\ \cos\omega t - \sin\omega t \end{pmatrix}.$$

(a) [**3 points**] What are the possible outcomes of a measurement of x component of the electron's spin at time t? Give the probability of each outcome.

(b) [**6 points**] Calculate the expectation values of spin operators $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ at time t.

(c) [**1 point**] What is the direction of the axis about which the spin expectation value is precessing, and what is the angular frequency of precession?