Student ID Number: ________

**PRELIMINARY EXAMINATION**

Department of Physics
University of Florida
Part D, January 7, 2015, 14:00–17:00

## Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

**University of Florida Honor Code:** We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

**DO NOT OPEN EXAM UNTIL INSTRUCTED**

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D1. **- Rinzler** The speed distribution of particles in an ideal gas that hit the walls of a container (which differs from the speed distribution of the particles in the bulk of the gas) is given by:

$$h(v)dv \propto v^3 e^{\frac{mv^2}{2k_B T}} dv$$

To answer the questions below, you may need the following integrals:

$$\int_0^\infty x^n e^{-ax^2} dx = \frac{\Gamma\left(\frac{n+1}{2}\right)}{2\left(a^{\frac{n+1}{2}}\right)}, \; n > -1; \; a > 0$$

$$\text{where } \Gamma(2) = 1! = 1, \rightarrow \Gamma\left(\frac{5}{2}\right) = \frac{3}{4}\sqrt{\pi}, \; \Gamma(3) = 2! = 2$$

(a) **[2 points]** Find the constant that makes $h(v)$ a normalized probability density function.

(b) **[2 points]** Find the average speed for this distribution.

(c) **[2 points]** Find the speed $v_{max}$ at which the probability is a maximum.

(d) **[2 points]** Show that the average energy for this distribution is $2k_BT$ (hint: as an ideal gas all the energy resides in kinetic energy). To get $2k_BT$ you will need to have gotten the answer to part (a) for the normalization constant. If you did not get an answer for part (a) take the solution here as far as you can keeping that as a constant.

(e) **[2 points]** The average energy for the bulk ideal gas is $\frac{3}{2}k_BT$, but in part (d) the average energy for the particles striking the container walls is $2k_BT$. Why is the latter greater?

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D2. **- Hebard**

Physicists often work with data sets where an independent variable, say resistance $R$, is measured as a function of a dependent variable, say temperature $T$. If the theory describing the temperature dependence is given by the relation,

$$R(T) = R_0 T^{\alpha} \tag{1}$$

where $R_0$ and $\alpha$ are unknown constants, then the easiest way to find these unknown constants is to use linear regression algorithms which will find best fit values of $R_0$ and $\alpha$ by minimizing the sum of the squares of each data point from its best fit value. For the above example this is done by taking the logarithm of both side of Eq. 1, obtaining,

$$\log R(T) = \log R_0 + \alpha T. \tag{2}$$

and then using regression to get the best fit values of the *intercept*, $\log R_0$, and the *slope*, $\alpha$. The best fit values of the unknown parameters are $R_0 = 10^{intercept}$ and $\alpha = slope$. Most linear regression programs will provide error estimates (standard deviations) on the fit parameters *slope* and *intercept*.

The electrical properties (current $I$ versus voltage $V$) of a metal-semiconductor interface are described by the nonlinear equation:

$$I(V,T) = I_S(T)\left[\exp(eV/\eta k_B T) - 1\right] \tag{3}$$

where the electronic charge $e = 1.6 \times 10^{-19}$ C, the Boltzmann constant $k_B = 1.38 \times 10^{-}23$ J/K, and $\eta$ is an ideality constant which can be as small as unity for an ideal junction obeying thermionic emission theory. The saturation current $I_S(T)$ is independent of $V$ and is described by the equation

$$I_S(T) = I_{S0}\exp(-e\Phi_{SBH}/k_B T) \tag{4}$$

where $I_{S0}$ is a constant and $\Phi_{SBH}$ is the Schottky barrier height indicated in the energy level diagram of Fig. 1.

(a) [**1 point**] Suppose you are given a multidimensional data set where the current-voltage characteristics described by Eq. 3 have been determined for 100 equally spaced temperatures ranging from room temperature (300K) down to 4K. Temperature and voltage are the independent variables. For positive voltages the junction is forward biased with a current which increases exponentially with voltage providing $eV >> \eta k_B T$. Determine the crossover voltage (numerical value) for an ideal junction at $T = 300$ K where this exponential dependence begins to dominate.

(b) [**1 point**] For $I - V$ data at each fixed temperature assume that your forward bias voltage is high enough to be in the exponentially dominated region so that the -1 in Eq. 3 can be ignored. Linearize Eq. 3 and make a sketch of how the data will look. Be sure to label each axis in your sketch.

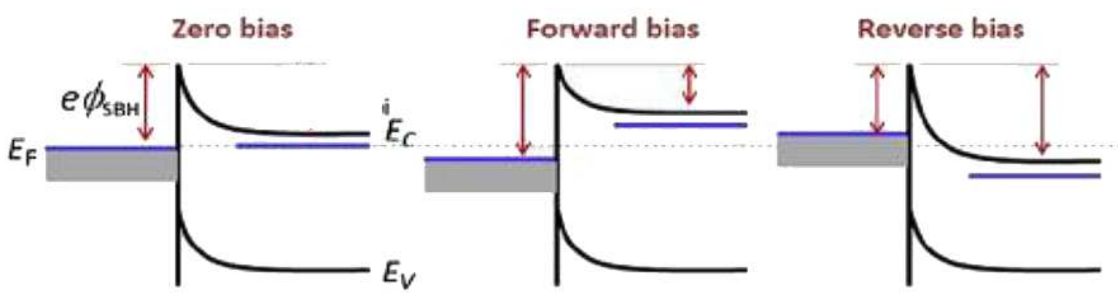
(c) [**2 points**] Use your "linear regression program" to determine the $slope_T$ and $intercept_T$ of each plot written in terms of the variables/parameters of Eq. 3 and Eq. 4. The subscript indicates that the temperature is held constant for each of the 100 data pairs.

(d) [**2 points**] Describe and sketch how you would plot the 100 values of $intercept_T$ versus an independent variable to arrive at a linear plot. Again, use your "linear regression program" to determine $slope_{D1}$ and $intercept_{D1}$.

(e) [**2 points**] Repeat Part D for $slope_T$ to determine $slope_{E1}$ and $intercept_{E1}$

(f) [**1 point**] From the four regression fit parameters found in Parts D and E, write expressions for your best fit estimates of the unknown parameters $\eta$, $\Phi_{SBH}$ and $I_{S0}$ in Eqs. 3 and 4. Box your answers.

(g) [**1 point**] Sketch the current $I$ versus voltage $V$ characteristics at a single temperature described by Eq. 3. Use linear scales and include both voltage polarities. Label your sketch at points of interest.




Schematic energy level diagram showing a metal/semiconductor interface at zero bias, forward bias ($V > 0$) and reverse bias ($V < 0$). $E_F$ is the Fermi energy of the metal, $\Phi_{SBH}$ is the Schottky barrier height, and $E_C$ and $E_V$ are respectively the conduction and valence bands of the semiconductor.

D3. **- Acosta**

The muon is an unstable particle that decays according to a random process with a mean lifetime in its rest frame of $\tau = 2 \times 10^{-6}$ s into an electron and two antineutrinos. Suppose that cosmic rays create muons in the upper atmosphere exactly 10 km above the surface of the Earth all with identical kinetic energies of 1000 MeV. The rest mass of the muon is approximately 100 MeV/$c^2$, where 1 MeV = $1.6 \times 10^{-13}$ J. For the questions below, please provide also a numerical answer.

(a) [**2 points**] Assuming only Newtonian kinematics, what is the speed of the muons?

(b) [**2 points**] Assuming only Newtonian kinematics, what fraction of muons traveling vertically downward reach the surface of the Earth before decaying?

(c) [**2 points**] Assuming relativistic kinematics, what is the speed of the muons?

(d) [**4 points**] Assuming relativistic kinematics, what fraction of muons traveling vertically downward reach the surface of the Earth before decaying?