Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, January, 2016, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

A1. **(Konigsberg)** A spin 1/2 electron with magnetic moment μ is placed in a uniform magnetic field $\vec{B} = B\hat{k}$ in the positive z-direction. The intrinsic spin of the electron is pointed along the positive x-direction at $t = 0$.

(a) [**2 points**] Write down the Schrödinger equation for the two component wave function for the electron at rest.

(b) [**3 points**] Find the corresponding time-dependent wave function.

(c) [**3 points**] Calculate the expectation values $< S_x(t) >, < S_y(t) >, < S_z(t) >$.

(d) Find the probability as a function of time that the intrinsic spin will be pointed along:

- [**1 point**] the positive z-direction
- [**1 point**] the positive x-direction

This maybe useful:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

$$| S_z^+ \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} | S_x^+ \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2}$$

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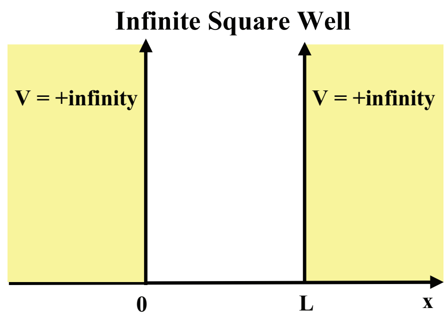
A2. **(Field)** Consider an electron with mass m_e confined within a one-dimensional infinite square well defined by

$$V(x) = 0 \text{ for } 0 < x < L$$
$$V(x) = +\infty \text{ otherwise.}$$



(a) **[2 points]** Using Schrödinger's equation, calculate the allowed stationary state eigenfunctions $\Psi_n(x)$, where the complete wavefunctions are given by $\Psi_n(x,t) = \Psi_n(x)e^{-iE_nt/\hbar}$. Normalize the eigenfunctions so that the probability of finding the electron somewhere in the box is one.

(b) **[2 points]** Show that the wavefunctions $\Psi_n(x,t)$ correspond to states with definite energy (*i.e.*, show that $\Delta E = \sigma_E = 0$).

(c) **[2 points]** Calculate the allowed energy levels, E_n, of the system. Express your answer in terms of the Compton wavelength of the electron, $\lambda_e = \hbar/(m_e c)$, and the rest mass energy of the electron, $m_e c^2$. What is the ground state energy (**in MeV**) for the case $L = \lambda_e$? (Note that $m_e c^2 = 0.511$ MeV.)

(d) **[4 points]** Suppose the electron in this infinite square well has a wave function at $t = 0$ which is given by

$$\Psi(x,0) = \frac{2}{\sqrt{L}} \sin(2\pi x/L)\cos(4\pi x/L).$$

If you measure the energy of this particle, what are the possible values you might get, and what is the probability of getting each of them? What is the expectation value of the energy for this state (*i.e.*, average energy)? What is $\Psi(x,t)$ and what is the expectation value of x, $\langle x \rangle$, for the state $\Psi(x,t)$? Does $\langle x \rangle$ depend on time?

Useful Math

Trigonometric Relations:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$
$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$
$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

Indefinite Integrals:

$$\int \sin x \cos x dx = \frac{\sin^2 x}{2}$$
$$\int x \sin x dx = \sin x - x\cos x$$
$$\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4}$$
$$\int x \sin^2 x dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$
$$\int x^2 \sin^2 x dx = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin 2x - \frac{x\cos 2x}{4}$$
$$\int x \cos x dx = \cos x + x \sin x$$
$$\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4}$$
$$\int x \cos^2 x dx = \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8}$$
$$\int x^2 \cos^2 x dx = \frac{x^3}{6} + \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin 2x + \frac{x\cos 2x}{4}$$

Definite Integrals:

$$\int_0^{\pi} \sin mx \sin nx dx = \frac{\pi}{2}\delta_{mn}$$
$$\int_0^{\pi} \cos mx \cos nx dx = \frac{\pi}{2}\delta_{mn}$$
$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$
$$\int_0^{\infty} x^{n-1} e^{-x} dx = \Gamma(n)$$

$$\Gamma(n+1) = n\Gamma(n) \qquad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

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A3. **(Cheng)** Two identical particles, each with mass M and spin $\frac{1}{2}$, are described by the Hamiltonian,

$$\hat{H} = -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + \frac{1}{2}M\omega^2(x_1^2 + x_2^2).$$

The two particles are in a state described by the wave function,

$$\psi(x_1, x_2) = \sqrt{\frac{2}{\pi}}\frac{(x_1 - x_2)}{a^2}\, e^{-(x_1^2+x_2^2)/2a^2}\, |\chi\rangle. \tag{3}$$

Here, $|\chi\rangle$ is a vector in a four-dimensional spin vector space, where $|m_1, m_2\rangle$ is a basis state for this space satisfying $\hat{S}_{jz}|m_1, m_2\rangle = m_j\hbar|m_1, m_2\rangle$ for $j = 1$ and 2, and $a = \sqrt{\hbar/M\omega}$.

(a) **[4 points]** What are the three mutually orthogonal and physically acceptable spin vectors $|\chi\rangle$ in the state described by Eq. (3)? Express your normalized answers in the basis $\{|m_1, m_2\rangle\}$.

(b) **[3 points]** Find the total energy of the system in the state described by Eq. (3).

(c) **[3 points]** Write down another stationary state of the system that is orthogonal to, but degenerate in energy with, the three states in part (a). Specify both the spatial and spin parts.