Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, January, 2016, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

    (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

    (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

    (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

    (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

    (e) Each problem is worth 10 points.

    (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. **(Takano)** Consider a molecule containing three $S = 1/2$ spins, $\vec{S}_1$, $\vec{S}_2$, and $\vec{S}_3$. The Hamiltonian of this molecule in magnetic field $\vec{B}$ is $\mathcal{H} = J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_1) - a(\vec{S}_1 + \vec{S}_2 + \vec{S}_3) \cdot \vec{B}$, where J and a are positive constants. The eigenstates $|S_{total}, m\rangle$ and eigenvalues of the Hamiltonian are given in the table below, where S_{total} and m are the quantum numbers for the total spin, $\vec{S}_{total} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$, and $S_z = S_{1,z} + S_{2,z} + S_{3,z}$. z is the direction of the magnetic field.

Eigenstate	Eigenvalue
$\|\frac{3}{2}, -\frac{3}{2}\rangle$	$\frac{3}{4}J + \frac{3}{2}aB$
$\|\frac{3}{2}, -\frac{1}{2}\rangle$	$\frac{3}{4}J + \frac{1}{2}aB$
$\|\frac{3}{2}, \frac{1}{2}\rangle$	$\frac{3}{4}J - \frac{1}{2}aB$
$\|\frac{3}{2}, \frac{3}{2}\rangle$	$\frac{3}{4}J - \frac{3}{2}aB$
$\|\frac{1}{2}, -\frac{1}{2}\rangle$	$-\frac{3}{4}J + \frac{1}{2}aB$
$\|\frac{1}{2}, \frac{1}{2}\rangle$	$-\frac{3}{4}J - \frac{1}{2}aB$

The last two states, with $S_{total} = 1/2$, are both doubly degenerate, whereas the others are non-degenerate.

(a) [**5 points**] Give an expression for the thermal average $\langle S_z \rangle$ of S_z as a function of temperature at $B = \frac{3}{4}J/a$. Note that at this field, the $|\frac{1}{2}, -\frac{1}{2}\rangle$ states and the $|\frac{3}{2}, \frac{3}{2}\rangle$ state happen to have the same energy.

(b) [**3 points**] In the high-temperature limit, write the expression to first order in J/k_BT.

(c) [**2 points**] In the low-temperature limit, write the expression up to the first non-constant term.

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C2. **(Hamlin)**

(a) [**4 points**] The heat of melting (latent heat) of ice is about 300 J/g. At one atmosphere, ice (of course) melts at 273 K. One gram of water has a volume of 1 cm^3, while one gram of ice has a volume of about 1.1 cm^3. If the pressure is increased by 1 MPa $= 10^6$ $\mathrm{N/m^2}$, by approximately how much will the melting point of ice change. (Make sure to state whether the change you calculated represents an increase or decrease in the melting point.)

(b) [**3 points**] An electrically operated heat pump delivers 1200 J per second to the interior of a house at 300 K. It takes heat from the exterior of the house at a temperature of 250 K. What is the minimum possible electrical power consumption of the heat pump?

(c) [**3 points**] Two identical plates of copper are brought into thermal contact. Initially, one is at 100°C and the other at 0°C. What is the total change in entropy from the initial contact until the two blocks are in thermal equilibrium with each other? (You may assume that the heat capacity of each block, c, is constant over this temperature range, and you may neglect volume changes.)

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DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
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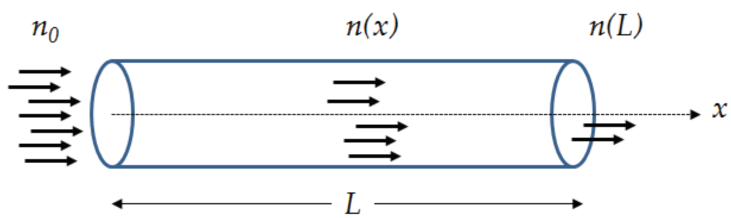
C3. **(Hagen)** The eye uses specialized cells called *rods* to detect photons. Each rod is a cylinder of length L and contains a large quantity of a protein that absorbs light. A photon that enters the front end of the rod travels in a straight line until it is either absorbed by the protein or exits the rear end. Every photon absorbed in the rod leads to an electrical pulse (signal) that is detected by the eye.



(a) [**2 points**] Suppose that n_0 photons enter the front face of the rod cell (at $x = 0$) and travel along the rod axis ($x-$axis). Let μ be a constant that describes the light absorbing property and concentration of the protein: Over each small distance dx, a fraction μdx of the traveling photons are absorbed (and hence detected). Consequently the number $n(x)$ of unabsorbed photons at any position x along the axis of the rod cell decreases as a function of x.

By considering the fraction of photons that are absorbed with each dx, find an expression for $n(x)$ in terms of μ, n_0 and the distance x that the light has traveled inside the rod.

(b) [**2 points**] From your result above, find n_a, the average number of photons that are absorbed (and thus detected) by a rod of length L, if n_0 photons are incident. Sketch n_a as a function of L.

(c) [**1 point**] You have found n_a, the total "signal" that is generated by the rod. Now consider the "noise". Suppose that the light-absorbing protein in the rod occasionally generates false, spontaneous signals (where no photon was absorbed). False signals are random events and occur at a very low rate of γ false signals per second, per unit length of the rod. On average, what mean number $\langle n_1 \rangle$ of such signals is expected in one second in a rod of length L?

(d) [**3 points**] If n_1 were absolutely predictable then the eye could learn to ignore it. Unfortunately n_1 is subject to statistical fluctuation. State the name of the probability distribution that governs n_1. From the properties of that distribution, use the mean $\langle n_1 \rangle$ that you calculated above to find σ_{n_1}, the standard deviation of n_1. Give σ_{n_1} in terms of γ and L. This is the "noise" level in the rod cell.

(e) **[2 points]** Now express the "signal to noise ratio", n_a/σ_{n_1} as a function of L, γ, μ, and n_0. Sketch this ratio as a function of rod length L. State clearly whether it has a maximum at any finite value of L. That is, is there a finite rod length that provides optimal signal-to-noise ratio for the detection of incoming photons?