Turbulence in classical and quantum fluids

IV. Classical turbulence experiments in cryogenic helium
Rayleigh-Benard Convection

Control parameters for convection

\[ \text{Ra} = \frac{g \alpha \Delta T H^3}{\nu \kappa} \]
\[ \text{Pr} = \frac{\nu}{\kappa} \]
\[ \Gamma = \frac{D}{H} \]

\( \alpha \): fluid thermal expansion coefficient
\( \nu \): fluid kinematic viscosity
\( \kappa \): fluid thermal diffusivity
Thermal boundary layers at the upper and lower walls are highly stressed regions giving rise to “plumes.”

The temperature gradient is all at the wall! At high Ra in experiments the boundary layer can be of order 100 micrometers!

At sufficiently high Ra: Self-organization of turbulent convection and generation of a coherent “mean wind”

• coupling of top and bottom boundary layers due to wind
• importance of thermal conditions on sidewalls in presence of wind
• ability of heating plates to supply needed rate of plume formation
Inside the Black Box:

Global heat transfer: Nusselt number

\[ \text{Nu} \equiv \frac{\text{actual heat flux}}{\text{heat flux if by conduction at same } \Delta T} = \frac{qH}{k_f \Delta T} \]

- \( q = \text{applied heat flux corrected for sidewall conduction} \)
- \( k_f = \text{fluid thermal conductivity} \)

- **β = 1/3**:
  - Boundary layers (uncoupled) important (Priestley, Howard, Malkus, 1954); rigorous in limit of infinite Prandtl number (Constantin and Doering 2001).

- **β = 1/2**:
  - Kraichnan (1962) for moderate Pr (with logarithmic corrections)

Recent treatment: Lohse and Grossmann (2002)
Plausibility of scaling exponent $1/3$

$$\text{Nu} = \frac{qH}{k_f \Delta T}$$

At very high $Ra$ the temperature gradient is all at the wall, across boundary layers of thickness $\delta$

1. Heat flux across boundary layers: $q = \frac{k_f \Delta T}{2\delta}$

2. Rayleigh defined on the boundary layers:
   $$Ra_\delta = \frac{g \alpha \Delta T \delta^3}{\nu \kappa}$$
   and assume that $Ra_\delta$ reaches marginal stability value $Ra_c=1100$ (Chandrasekhar, slip/no slip)

3. Using (1) for $q$: $\text{Nu} = \frac{1}{2} \left( \frac{H}{\delta} \right)$

4. Then from (2): $\delta = \left( \frac{2Ra_c \nu \kappa}{g \alpha \Delta T} \right)^{1/3}$

5. Substituting $\delta$ from (4) into (3) and using (2):
   $$\text{Nu} = \left[ \frac{Ra}{16Ra_c} \right]^{1/3} \sim 0.04 Ra^{1/3}$$
Plausibility of scaling exponent 1/2

Convert gravitational potential energy into turbulent kinetic energy

\[ \Delta \rho g H \sim \rho u^2 \]

\[ u \sim \sqrt{\alpha \Delta T g H} \]

From the equations for the mean temperature difference the transport of heat by turbulent fluctuations is

\[ q' = \rho C_p u \theta \]

involving the correlation between vertical velocity fluctuations and fluctuations in temperature which we take to be \( \theta \sim \Delta T \)

Assuming that the contribution of molecular transport can be ignored (essentially no diffusive boundary layers) we can set

\[ \rho C_p u \theta \sim Nu \cdot \frac{k \Delta T}{H} \]

Using the scales for vertical velocity and temperature fluctuations we then have

\[ Nu \sim \sqrt{\frac{g \alpha \Delta T H^3}{\kappa^2}} = \mathcal{RaPr}^{1/2} \]
All this occurs at high $Ra$—how to get there?

12 decades of $Ra$ possible, but which 12 depends mostly on $H$. The shaded region gives an approximate area of operation with the P-T plane.

\[ Ra = g \cdot \left( \frac{\alpha}{\nu K} \right) \cdot \Delta T \cdot H^3 \]

**Cgs units**

4.4 K, 2 mbar:  
\[ \frac{\alpha}{\nu K} = 5.8 \times 10^{-3} \]

5.25 K, 2.4 bar:  
\[ \frac{\alpha}{\nu K} = 6.5 \times 10^9 \]

**Compare:**

Water:  
\[ \frac{\alpha}{\nu K} \sim 10 \]

Air:  
\[ \frac{\alpha}{\nu K} \sim 0.1 \]
A large cryogenic convection apparatus

Cryocooler

Pressure Relief and Sensing

Liquid Nitrogen Reservoir

Top Plate (Fixed Temperature)

Cell Fill Tube

Multilayer Insulation

L

T

h

termal

S

hields

\[ H_{(\text{maximum})} = 1 \text{ meter} \]

\[ D = 0.5 \text{ meter} \]
Facilities located at Elettra Synchrotron Laboratory, Trieste
Corrections due to finite conductivity of the plates

Verzicco (2004)

\[ \text{Nu} = F(X)\text{Nu}_{\text{inf}}, \text{ where } \text{Nu}_{\text{inf}} \text{ is achieved with "ideal" plates of 'infinite' thermal conductivity} \]

where

\[ X \equiv \frac{R_f}{R_p} = \frac{k_p H}{k_f \text{Nu} \cdot e} = N_{\text{Biot}}^{-1} \]

\[ F(X) = 1 - e^{-\frac{k_p}{4N_{\text{Biot}}}N_{\text{inf}}^3}. \]

\( k_p = \text{thermal conductivity of plates} \)
\( k_f = \text{thermal conductivity of fluid} \)
\( e = \text{thickness of the plates}. \)

Correction for ordinary fluid/plate combinations is an order of magnitude larger at only moderate Ra!
Sidewall conduction effects


Using the correction proposed by Roche, et al.
Measurements of Turbulent Heat Transfer

Conductivity enhancement by a factor of 20,000

Log-log plot of the Nusselt number versus Rayleigh number

\[ \Gamma = \frac{1}{2} \]

\[ \text{Nu}_{\text{corr}} = 0.088 \text{Ra}^{0.32} \]


A coherent “mean wind”…. …and an aspect ratio unity cell to measure it.

(from L. Kadanoff, *Physics Today*, August 2001)
An aspect ratio unity cell for enhancing the mean wind

250-micrometer NTD-doped Ge sensors are placed in various positions in the flow.

Stabilization: $10^5$ turn-over times of the wind

Run times: $10^4$ turn-over times of the wind
Note a **crude** estimate for the circulation frequency from correlation times:

\[
\frac{V_M}{4H} = \frac{7 \text{ cm/s}}{200 \text{ cm}} \approx 0.035 \text{ Hz}
\]

\[
\omega_p \approx \frac{2\pi}{\omega_p}
\]

\[
\frac{\omega_p H^2}{\kappa} \equiv \frac{V_M H}{\kappa} \equiv Pe \approx Re
\]

From inset we get \( \text{Re} \sim \text{Ra}^{1/2} / \text{Pr} \)

Note: the velocity is almost independent of Ra—nearly all variation of Re comes from the diminishing kinematic viscosity.
The mean wind and its reversals

Segment of continuous 5.5-day time series.

Geomagnetic polarity reversals: range of time scales $\sim 10^3$-$10^5$ years.

Glatzmaier, Coe, Hongre and Roberts Nature 401, p. 885-890, 1999
Medium energy solar flares owe their duration to turbulent convective motions in the convective zone of the sun which shuffle footprints of the magnetic coronal loops (Parker, 1994).

Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI)

Comparison of the duration of single-direction wind in RBC experiments to the duration of solar flares observed by RHESSI
Possible to measure “eddy diffusivity” in cryogenic environment


Solution to diffusion equation:

\[ T(\xi, \omega) = T_{B_0} \exp(-z/\delta_S) \]

\[ \delta_S = \sqrt{\frac{2 \kappa_{\text{eff}}}{\omega}} \]

Small sensor at mid-height

Wind signal amplitude, deg K

frequency, Hz

Ra=3.5x10^9

\[ \text{Nu (from } \kappa_{\text{eff}}) = 96.5 \]

\[ \langle \text{Nu}(t) \rangle = 98.6 \]

\[ \text{Nu (JJN & KRS, 2003)} = 98.8 \]

\[ (T_{B_0})_{\text{rms}} = 0.00706 \text{ K} \]
Rotation about vertical axis

Rotation rate: \( \Omega_d \) (rad/s)

Dimensionless rotation rate: \( \Omega \equiv \left( \frac{2 \Omega_d H^2}{\nu} \right) \)

Taylor number: \( Ta = \Omega^2 \)

Convective Rossby number: \( Ro = \left[ \frac{Ra}{PrTa} \right]^{1/2} \)

Experimental parameters:

\( 10^{11} < Ra < 4.3 \times 10^{15} \)

\( 10^{11} < Ta < 3.0 \times 10^{15} \)

\( 0.4 < Ro < 1.6 \)

\( 0.7 < Pr < 5.9 \)

Apparatus placed on rotating platform controlled by optically encoded motor.

All electronics onboard, including computer.
Heat transport with steady rotation at large Ra, Ta

\[ Ra = 4.3 \times 10^{15} \quad Pr = 5.9 \text{ (comparable to water)} \]

The effect of steady rotation is nearly zero in agreement with recent work of Zhong, et al (2009), but…

…the “nearly zero” effect is measurably negative, i.e. increasing Taylor numbers and decreasing Rossby numbers lead to a reduction in the heat transport.

von Karman swirling flow (otherwise known as the “French washing machine”)

Largest cell: 20 cm in diameter and 13.1 cm tall (measured between the disks).

Must be considered a “table-top” experiment.

Enclosed helium gas: pressure from 0 to 6 atm and temperature from 4.2 to 8 K.

The Reynolds number, defined here as $Re = \frac{\Omega R^2}{\nu}$, where $\Omega$ is the angular velocity of the disks, varied from $10^4$ to about $10^7$. 
The quantity most often measured in these experiments is a component of the velocity in the direction of the disk rotation. This can be quite strong near the disks due to the entrainment of the fluid by the fins.

It is conventional to interpret fluctuations measured as a function of time entirely as spatial fluctuations, the connection being the sweeping by the mean velocity at the measurement point. This is the so-called Taylor’s frozen-flow hypothesis.

The temperature sensitivity of the bare wire is 0.3 W/K. Time response from 3 to 16 ms. Gold/silver evaporated elsewhere for electrical contact.

Fig. 2. Cryogenic hot wire probe, showing the active region, \( \ell_w = \mathcal{O}(7 \, \mu\text{m}) \). After Tabeling et al.\textsuperscript{34}
While hot-wire measurements at room temperature have been routine for some 60 years, this is not the case at low temperatures. First, any usable probe has to retain a large sensitivity at low temperatures.

Second, because of the high Reynolds numbers in cryogenic flows, the corresponding Kolmogorov scale will be quite small in magnitude (of the order of 1μm in these experiments) and the measurement transducers must be correspondingly small.

The active area of the sensor was a cylinder of diameter and length equal to 7μm. Even this small size is much larger than the Kolmogorov scale at the highest $Re$ achieved.

Longitudinal increments of velocity were calculated on different scales. These increments are defined as $\Delta V_r = V(x+r) - V(x)$, where $V$ is the local velocity, and the separation length $r$ is determined from the measured separation time using the Taylor frozen flow hypothesis.
As the scales decrease toward the Kolmogorov dissipation length, the increments display more intermittency, indicated by the presence of long “tails” in the probability distribution.

Intermittency refers to the existence of “patchiness” in the spatial distribution of small scales and leads a departure of the random velocity field from the self similarity assumed by Kolmogorov’s phenomenological theory.

It is a general feature of turbulent flows related to the relatively frequent and large excursions of the data (e.g., velocity increments) from the mean. Intermittency renders Gaussianity inapplicable to turbulence broadly speaking.
The intermittency as a relative measure of the distribution of tails can be partially quantified by the factor

\[ F(r) = \frac{<\Delta V_r^4>}{<\Delta V_r^2>^2}. \]

Taking the limit of this quantity as the separation distance approaches zero yields the flatness factor \( F \) of the derivative. Clearly, the authenticity of the derivative obtained from experimental data depends on whether the velocity has been resolved and sampled adequately.

The data at higher Reynolds numbers have been obtained in the atmospheric flows where resolving Kolmogorov scale (of the order of 0.5 mm) from conventional hot wires presents no problem.

One of the surprises from helium experiments (below) is that the flatness of the measured “derivative” decreases with increasing Reynolds number beyond a certain critical value, $R_\lambda = 700$ (here the internal microscale Reynolds number is roughly equivalent to the square root of the Reynolds number based on the large scales).

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![Graph](image)

**Fig. 4.** Flatness of velocity derivatives versus $R_\lambda$ (after Emsellem et al.38). The solid and open symbols refer to two different experimental cells, the larger one (open symbols) having a radius roughly 3 times that of the other (solid symbols).
If this is true, the measurements have revealed new physics that was missed earlier…

There is some realization recently that the Kolmogorov scale is, in fact, a fluctuating quantity at high Re, and that scales significantly smaller than the mean value must exist (Schumacher, et al). In fact, the higher the order of the moment and the higher the Reynolds number, the more stringent is the required resolution in measurement and simulations.

Discerning whether helium experiments at such high Re have actually revealed new physics will require improvements in sensor technology, which is common thread for all low temperature experiments.
In pipe flows, disturbances of finite amplitude are responsible for the transition to turbulence. Reynolds noticed as much when he reported that the transition was delayed to higher values of $Re$ when a particularly smooth entrance region of the pipe was used. He found that the transition to turbulence typically occurred above a critical value of about 2000 for the ratio of $UD/\nu$, where $U$ is the average (or bulk) velocity, $D$ is the pipe diameter, and $\nu$ is the kinematic viscosity a combination then named after him.
“Super-laminarized” flow

The ability to control the transition to turbulence could be a real advantage, especially for oil pipelines which are usually operated always in the much less efficient turbulent state to avoid large fluctuations due to the transition from laminar flow to turbulent flow.

State-of-the-art pipe flow at Univ. of Manchester (T. Mullin)

![Diagram of pipe flow](image)

Fig. 2. (Color on-line) Schematic of the constant mass flux pipe facility. The pipe, the reservoir, and the piston are up to scale. The temperature of the laboratory was controlled to $\pm 1^\circ C$ at a mean temperature of $20^\circ C$. 
Shear stress at the wall: Friction factor

For pipe flow, as long as entrance effects, roughness, and temperature variations are small, dimensional analysis indicates that the friction factor \( \lambda \) is only a function of the Reynolds number \( Re \). That is,

\[
\lambda = f(Re)
\]

where

\[
\lambda = \frac{-(dP/dx)D}{\frac{1}{2}\rho U^2} \quad \text{and} \quad Re = \frac{UD}{v}.
\]

Here \( f \) denotes a functional relationship, \( dP/dx \) is the pressure drop per unit length, \( D \) is the diameter of the pipe, \( \rho \) is the fluid density, \( v \) is the kinematic viscosity, and \( \overline{U} \) is the flow velocity averaged over the cross-sectional area of the pipe.

Here, the friction factor \( \lambda \) and shear stress at the wall \( \tau \) are related by \( \tau = \lambda (1/2\rho U^2) \).
The flow through a cryogenic pipe operated by C. Swanson, G. Ihas and some others at Univ. of Oregon (shown on left where the fluid is helium I) is generated by a controlled compression of a metal bellows attached to the mouth of the pipe of diameter 4.7 mm.

The pressure gradient generated by the flow is measured near the exit of the pipe with two static pressure taps.

Capacitative pressure transducer

The gauge operates equally well in liquid helium and in room temperature gas and can resolve pressure differentials down to 0.01 Pascal.
Table-top experiments for measuring friction factor

Oregon measurements

Comparison of sizes of water and helium apparatus at Oregon

Nikuradse from Schlichting’s Boundary layer theory
“Superpipe” in Princeton

- $R \approx 6.46 \text{ cm}$
- $L/D = 200$
- air at 200 atm
- $Re \approx 5000 - 3 \times 10^7$
Comparison of friction factor data (plusses) from Princeton super-pipe and cryogenic experiment (circles)

Not a table top experiment…
Princeton Superpipe: weight = 35 tons

Other robust, small scale probes are needed for local measurements in the case of helium