

Laboratory #8
Size of a molecule

A molecule, as you will probably remember from Chemistry, is the smallest possible piece of a compound. For instance, a molecule of water consists of two hydrogen atoms and one oxygen atom. In this laboratory we will estimate the size of a particular (rather large) molecule. We put a little drop of stuff called oleic acid on top of a pan of water. The water has some chalk dust sprinkled on top of it. The oleic acid spreads out, sort of like an oil slick, until it forms a thin film on top of the water which is just one molecule thick; and the thickness of the slick is just the size of the molecule! We can measure the area of the slick because the stuff pushes the chalk dust out of the way. And the thickness is just the original volume of oleic acid divided by the area of the slick. Sounds pretty straightforward.

A difficulty with this experiment is that a drop of pure oleic acid would spread out enough to cover a good sized swimming pool. Since we don't have a pan of water that big, we want to dilute the stuff a great deal first with something that won't effect the size of the slick. Alcohol is ideal for this—it mixes with the oleic acid, and it quickly evaporates once the drop hits the water, so it won't add to the size of the slick. We have already diluted the mixture to 1 part in 400 being oleic acid.

We need to know how much oleic acid there is in a drop of our mixture. So what is the volume of a drop? We take the dropper and count the number, n , of drops needed to fill up a graduated cylinder to the 1 cc mark (that is chemistry slang for 1cm^3). Then the volume, V , of oleic acid in a drop of solution is

$$V = (400n)^{-1}\text{cm}^3,$$

the $1/400$ comes from the dilution of the mixture.

Now, to measure the area of the slick we put the drop of stuff in the middle of the pan of water and watch it form a nice circle—or at least a shape resembling a circle. We can measure the approximate diameter and halve that to get the radius, R . And the area is

$$A = \pi R^2.$$

Finally, the thickness, t , of the slick is

$$t = V/A,$$

and this is also the approximate size of the oleic acid molecule.

But that's not all. There is a bonus in today's laboratory, because we can also estimate Avogadro's number, \mathcal{N} . Avogadro's number is what relates the mass of a molecule to everyday units. The chemical formula for oleic acid is $\text{C}_{17}\text{H}_{33}\text{COOH}$; this gives a molecular weight of about 282 (this is really close to the total number of neutrons and protons in the molecule.) By the definition of Avogadro's number, 282 gm of oleic acid has Avogadro's number worth of molecules in it. Now the density ρ of oleic acid is about the same as that of water, $\rho = 1\text{gm}/\text{cm}^3$. So the mass of the oleic acid in the drop is $M = \rho \times V$ grams. Therefore, the number of molecules, N , in M grams is

$$N = \frac{\mathcal{N}M}{282\text{ gm}},$$

so

$$\mathcal{N} = \frac{N282 \text{ gm}}{M},$$

and we can estimate the number of molecules in our drop. We measured both the area of the slick and the typical size of the molecule, t , already. If the typical size is t then the typical volume of one molecule is t^3 , as if the molecule were a cube. So, the number of molecules N must be the total volume V divided by the volume of each molecule t^3 :

$$N = \frac{V}{t^3}$$

A molecule of oleic acid is not really shaped like a cube, but our estimates are pretty rough already so we don't care. Putting these last few formulas together, we get for Avogadro's number

$$\mathcal{N} = \frac{282 \text{ gm}A}{Mt^2},$$

or simpler yet, because $M = \rho \times V = \rho At$

$$\mathcal{N} = \frac{282 \text{ gm}A}{\rho At^3} = \frac{282 \text{ gm}}{\rho t^3}.$$

Recall that the density of oleic acid is about $\rho = 1 \text{ gm/cm}^3$. So if we measure t in cm then

$$\mathcal{N} = \frac{282 \text{ gm}}{\rho t^3} = \frac{282}{t^3}.$$

This measured value of Avogadro's number is usually in error by a factor of about 100 because the oleic acid molecule is actually real long and skinny and, when it is on top of water, all of the molecules stand straight up.

In this experiment it is important to be sure to make all measurements in centimeters.

In this one laboratory we have measured a very big number, Avogadro's number, and a very small one, the size of a molecule.