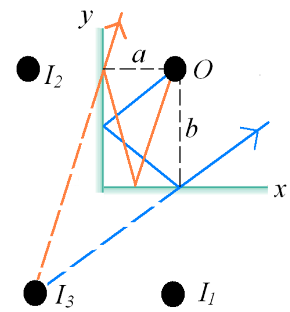
Instructor: Tomoyuki Nakayama Tuesday, July 12, 2011

PHY 2005 Applied Physics 2 - Summer 2011
Solutions to Suggested Homework Problems (Chapter 26)

Questions and Exercises



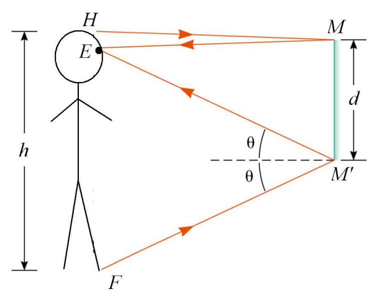
3. Two images I_1 and I_2 shown in the figure are produced by a single reflection from the horizontal or vertical mirror. Third image I_3 is formed due to double reflections from horizontal and vertical mirrors. The object and three images make a rectangle. If the position of the object is given by $x = a$ and $y = b$, the orange and blue rays in the figure, after two reflections, are expressed as $y - b = (2b/a)x$ and $y = (b/(2a))(x - a)$
These rays intersect at $x = -a$ and $y = -b$. You don't need to be able to prove it mathematically, but you should be able to sketch properly and find the position of I_3. *Sketching is very important* for solving problems on geometrical optics!

7. When the right wheel enters sand, it slows down. As a result, the left wheel rolls faster than the right wheel, and the two wheel device turns clockwise. This continues until the left wheel enters sand. After that the device rolls straight in the direction with a smaller angle with respect to the normal to the interface. This is exactly the same as light encountering an interface between two media. If a light ray traveling in a medium with a lower index of refraction enters a medium with a higher index of refraction, it bends toward the normal. The axle of the device corresponds to the wave front.

11. I am not sure if kittens are really frightened, but I think it is more frightening if their images are inverted. Thus I say that the concave mirrors are more frightening. Note that the image is inverted only when the object is outside of the focal length.

12. Optically the two media are the same. So there is no reflection at the interface between them. To see an object, a light ray must be reflected at the surface of the object and then enter your eyes. Therefore, you cannot see the object if there is no reflection at the surface.

Problems



2. The light ray that starts from the top of your head and reach your eyes hits the mirror at the height midway between your head and eyes. Therefore, if the length of the mirror is minimum, the top of the mirror should be located at this point. Likewise, the ray that starts from your feet and reach your eyes hits the mirror at the height midway between your eyes and feet. Hence the bottom of the mirror must be located at this point. Since ΔHEM and $\Delta EM'F$ are both isosceles, the height of the mirror is $d = (1/2)HE + (1/2)EF = h/2 = 0.850$ m

7. The focal length of the mirror is $f = R/2 = 30$ cm. a) The image distance is obtained by the mirror equation: $1/p + 1/q = 1/f \Rightarrow q = (pf)/(p - f) =$ 40 cm. The positive q implies the reflected rays actually converge in front of the mirror. Therefore, the image is real. The magnification is $m = -q/p = 0.667$. Hence the image height is $h_i = mh_o = 1.33$ cm. Since hi is negative, it is inverted. b) We repeat the same procedures as a) $q = 48$ cm, $h_i = -1.2$ cm, inverted, real. c) $q = 120$ cm, $h_i = -6$ cm, inverted, real. Note that the image moves away from the mirror as the object gets closer to it. d) $q = -60$ cm, $h_i = +6$ cm. Since q is negative, the image is behind the mirror and it is negative. The positive h_i indicates that the image is upright. It is an important fact that *the image due to a concave mirror is real if the object is outside of the focal length and virtual if it is outside of the focal length.*

9. Now we have a convex mirror, thus the focal length is negative: f = -30 cm. a) Solving the mirror equation for q, we get $1/p + 1/q = 1/f \Rightarrow q = (pf)/(p - f)$ = -24 cm. Since it is negative, the image is in the back of the mirror and virtual. The height of the image is $h_i = mh_o$ = (-q/p) h_o = 0.4 cm. The positive height implies the image is upright. b) We repeat the same steps as we did for a). q = -21.8 cm, h_i = 0.545 cm, virtual, upright. c) q = -17.1 cm, h_i = 0.855 cm, virtual, upright. d) q = -12 cm, h_i = 1.20 cm, virtual, upright. Note that the *image due to a convex mirror is always behind the mirror and inside of the focal length. It is virtual, upright and smaller than the object.*

16. a) Since the image formed by the convex mirror is behind the mirror, the image distance is negative: q = -5 cm. The magnification is $m = -q/p$ = 0.417 and the image size is $h_i = mh_o$ = 0.167 cm.

18. The real image formed by the concave mirror is inverted. Thus the magnification is m = -4 = $-q/p$. This relates the image distance with the object distance as $q = 4p$. The mirror equation yields $1/p + 1/q = 1/f \Rightarrow 1/p + 1/(4p) = 1/f \Rightarrow p = (5/4)f$ = 37.5 cm. The image distance is $q = 4p$ = 150 cm.

25. Frequency is independent of media. Thus the frequency of the light in the glass is the same as that in vacuum. It is given by $f = c/\lambda_0 = 5.09\times10^{14}$ Hz. The velocity in the glass is $v = c/n = 1.94 \times10^8$ m/s. The wavelength is $\lambda = \lambda_0/n$ = 380 nm.

27. a) Applying Snell's law, we get $n_a\sin\theta_1 = n_w\sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}(n_a\sin\theta_1/n_w)$ = 45.0º b) Snell's law yields $n_a\sin\theta_1 = n_w\sin\theta_2 \Rightarrow \theta_1 = \sin^{-1}(n_w\sin\theta_2/n_a)$ = 27.1º

31. The critical angle is given by $n_g\sin\theta_g = n_o$. Therefore, the refractive index of the oil is n_o = 1.33. Note the total internal reflection occurs only when light rays travel from a medium with higher index of refraction to a medium with lower index of refraction.

32. The angle of refraction for the blue light is $\sin\theta_B = n_B\sin\theta_B \Rightarrow \theta_B = \sin^{-1}(\sin\theta/n_g)$ = 25.896º. Likewise, the angle of refraction for the red light is $\theta_R = \sin^{-1}(\sin\theta/n_R)$ = 26.138º. The angle between these two refracted rays is $\theta_R - \theta_B$ = 0.242º.

37. To see the fisherman on the shore, the fish needs to look at an angle so that the rays travel almost horizontal after refraction. Snell's law yields $n_w\sin\theta_w = n_a\sin90º \Rightarrow \theta_w = \sin^{-1}(n_a/n_g)$ = 48.8º

40. The cat is inside of the focal length. Therefore, the image is virtual and upright. The image position is obtained by the mirror equation: $1/p + 1/q = 1/f \Rightarrow q = (pf)/(p - f)$ = -12 ft. The image size is $h_i = mh_o = (-q/p)h_0$ = 24 in. Note that the negative value of q implies the image is virtual, and the positive value of m implies the image is upright.

45. The critical angle of glass-air interface is $\theta_c = \sin^{-1}(n_a/n_g)$ = 45.6º. The angle of incidence on the right face is 50º (= 90 – (180 – 90 – 50)) and it is larger than θ_c, thus it undergoes total internal reflection. The angle of incidence at the bottom face is 15º (= 90 – (180 – 65-40)), so the ray emerges from this surface. The angle of refraction is $\theta = \sin^{-1}((n_g/n_a)\sin15º)$ = 21.2º.

47. The ray can escape from the water if the angle of incidence at glass-air interface is smaller than or equal to the critical angle there. The critical angle is $\theta_c = \sin^{-1}(n_a/n_g)$ = 38.7º. Since the angle of refraction at water-glass interface and the angle of incidence at glass-air interface are alternate angles, they are equal. Snell's law applied to the water-glass interface yields: $n_w\sin\theta = n_g\sin\theta_g \leq n_g\sin\theta_c \Rightarrow \theta \leq \sin^{-1}((n_g/n_w)\sin\theta_c)$ = 48.8º.