Instructor: Tomoyuki Nakayama Thursday, July 28, 2011

PHY 2005 Applied Physics 2 - Summer 2011
Solutions to Suggested Homework Problems (Chapter 28)

Questions and Exercises

1. As you rotate a tuning fork, the distances from your ear to the two prongs change, and the path length difference ranges from zero to the distance between two prongs. The wavelength of the sound is $\lambda = v/f = 0.034$ m. Since the destructive interference condition is $\Delta s = \lambda/2 + m\lambda$, as long as the distance between the two prong is comparable with $\lambda/2 = 1.7$ cm, alternate loud and weak sounds are heard.

5. Radio waves have longer wavelengths than the width of tree trunks and metal poles, so radio waves bend around the trunks and poles due to diffraction. Visible lights have much shorter wavelengths than the width of tree trunks and poles. Therefore, the diffraction does not occur and the trunks and poles cast shadows.

7. Two objects cannot be resolved if the angular separation is less than the critical angle that satisfies Rayleigh's criterion: $\sin\theta_C = 1.22\lambda/D$. Assuming the diameter of the pupil is $D = 5$ mm and the wavelength of visible light is $\lambda = 550$ nm, the critical angle is $\theta_C(\text{rad}) = \sin\theta_C = 1.22\lambda/D = 1.34 \times 10^{-4}$ rad. Note that $\theta_C(\text{rad}) \sim \sin\theta_C \sim \tan\theta_C$ for small angles. If the two headlights are separated by $d = 1.50$ m, the distance l between the eye and the car that corresponds to the critical angle is $d/l = \tan\theta_C \sim \theta_C(\text{rad}) \Rightarrow l = d/\theta_C = 1.1 \times 10^4 \sim 10.0$ km.

Problems

2. a) The path length difference is 0 and the two sources emit sound waves in phase. Therefore, they interfere constructively and the amplitude is $2A$. b) c) The path length is $\Delta s = |s_1 - s_2| = 90$ cm, which is equal to a half of the wavelength. Hence they interfere destructively. The amplitude is zero. d) The path length difference is 180 cm, which is equal to one wavelength of the sound. They interfere constructively and the amplitude is $2A$.

5. The condition for destructive interference is given by $\Delta s = (m + 1/2)\lambda$. As the frequency is increased, the wavelength decreases. The longest four wavelengths are $\lambda = 2\Delta s = 2.80$ m, $2\Delta s/3 = 0.933$ m, $2\Delta s/5 = 0.560$ m and $2\Delta s/7 = 0.400$ m.

7. The bright fringes are observed at points where the path length difference from the two slits is integer multiple of the wavelength. a) $0\lambda = 0$ b) $1\lambda = 546$ nm. c) $2\lambda = 1092$ nm. d) $3\lambda = 1638$ nm.

9. a) If the slit-screen distance is L and the slit separation is d, then the position of the m-th bright fringes with respect to the central fringe is given by $y_m = m\lambda L/d$. Plugging $m = 3$, we obtain $y_3 = 3\lambda L/d = 2.65 \times 10^{-2}$ m. b) The position of the m-th dark fringes with respect to the central fringe is given by $y_{m+1} = (m+1/2)\lambda L/d$. Plugging $m = 2$, we obtain $y_3 = 2.5\lambda L/d = 2.21 \times 10^{-2}$ m.

11. a) The separation between two neighboring slits is $d = 1/8000$ (cm) $= 1.25 \times 10^{-4}$ cm $= 1.25 \times 10^{-6}$ m, and the constructive interference condition for a diffraction grating is $d\sin\theta = m\lambda$. Solving the equation for λ and plugging $m = 2$, we get $\lambda = d\sin\theta/2 = 5.94 \times 10^{-7}$ m. b) If the angular position of the third-order maximum satisfies $\sin\theta = 3\lambda/d = 1.43$. Since the sine function cannot take a value larger than one, the third-order maximum is not observed.

12. We solve the constructive interference condition $d\sin\theta_m = m\lambda$ for the slit separation d. Plugging $m = 1$, we obtain $d = \lambda/\sin\theta_1 = 803$ nm $= 8.03 \times 10^{-5}$ cm b) The number of slits per centimeter is $n = 1/d(\text{cm}) = 1.24 \times 10^4$ (lines/cm).

17. The destructive interference condition for a single-slit experiment is $w\sin\theta = n\lambda$.

Therefore, the half-width angle of the central bright fringe is $w\sin\theta_c = \lambda \Rightarrow \theta_c = \sin^{-1}(\lambda/w) = 1.66º$.

20. The minimum angular separation that the telescope can resolve is $\theta_C(\text{rad}) \sim \sin\theta_C = 1.22\lambda/D = 6.57 \times 10^4$ rad. Considering the triangle subtended by the two objects on the moon, the minimum separation of the two object is $x = y\tan\theta_c \sim y\theta = 252$ m, where y is the earth-moon distance.

23. The wavelength of a light beam in a medium with refractive index n is $\lambda_n = \lambda/n$, where λ is the wavelength of the light beam in vacuum. a) $L = \lambda/2 = 316.5$ nm. b) $L = \lambda/(2n) = 238$ nm c)) $L = \lambda/(2n) = 214$ nm.

26. Since the ray that is traveling in air and reflected from the glass undergoes 180º phase shift. The constructive interference condition is $2t = (m+1/2)\lambda$. Plugging $m = 0, 1, 2, 3$, we obtain the first four separation for strongest reflection: $t = \lambda/4 = 110$ nm, $3\lambda/4 = 330$ nm, $5\lambda/4 = 550$ nm and $7\lambda/4 = 770$ nm.

32. Since oil has a larger index of refraction than water or air, the ray reflected at the oil-air interface undergoes a 180º phase shift, while there is no phase shift for the ray reflected from the oil-water interface. One phase shift is involved in the process. Therefore, the constructive interference condition is given by $2tn = (m+1/2)\lambda$, where n is the index of refraction of the oil. Since the thickness and wavelength are proportional to each other, as the color changes from red to yellow, to green, to blue and then to red, the thickness decreases. The difference between two thicknesses is $t_m - t_{m-1} = \lambda/(2n) = 203$ nm.

35. Only one phase shift occurs in the process. Therefore, the constructive interference condition is $2t = (m+1/2)\lambda$ or $m + 1/2 = 2t/\lambda$. As air fills the tube, 2770 fringes pass. This implies that after the air is filled in the tube, we have $2t = (m+1/2 + 2770)\lambda/n$. Combining this with the first equation, we get $(m+1/2)\lambda = (m+1/2 + 2770)\lambda/n \Rightarrow n = 1 + 2770/(m + 1/2) = 1 + 2770\lambda/(2t) = 1.000292$.