14

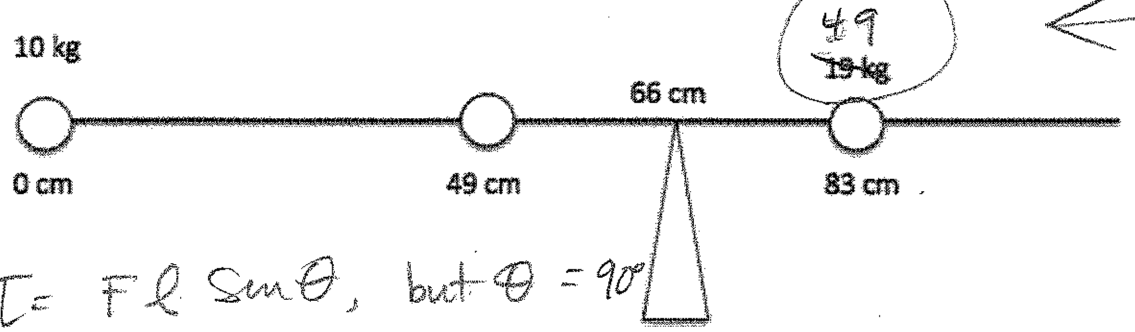
1. (X pts)

a. What conditions must be met for an object to be in equilibrium?

(2) $\Sigma F = 0$

(2) $\Sigma \tau = 0$

b. A meter stick has three weights attached to it, as shown in the image. We want the meter stick to be in a state of equilibrium when balanced at the 66 cm mark. Find the mass of the weight located at 49 cm that will balance this system.



$\tau = F l \sin\theta$, but $\theta = 90°$

(4) $\tau_{Left} = (10\,kg) g (66\,cm) + (X\,kg) g (66-49)\,cm$

(2) $\tau_{right} = (49\,kg) g (83-66)\,cm$

(2) let $\tau_{Left} = \tau_{right}$, cancel g from all terms

$660\,kg \cdot cm + X(17\,cm) = 833\,kg\,cm$

$X(17\,cm) = 173\,kg\,cm$

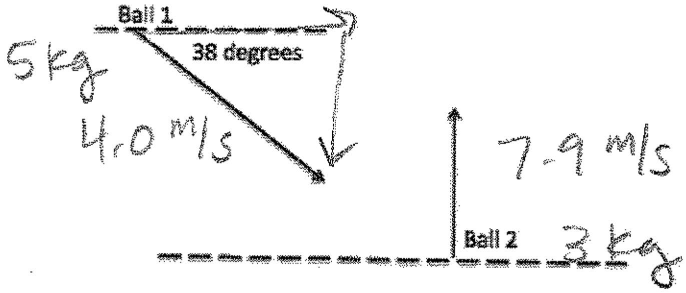
$X = 10.18\,kg$

(2)

(16)

2. (X pts) Ball 1, mass 5 kg, heads towards Ball 2 with an initial velocity of 4.0 m/s. The two balls collide and stick together. The mass of Ball 2 is 3 kg and its initial velocity is 7.9 m/s.

What is the final velocity of the system? Please remember to include directional information.



Cons of $\vec{p}$ in each direction

initial

(3) $P_x = (5\,kg)\,V \cos 38 = (5\,kg)(4.0\,m/s) \cos 38°$

(3) $P_y = (3\,kg)(7.9\,m/s) - 5\,kg\,(4.0\,m/s) \sin 38°$

final

(3) $p_x = (5+3)\,kg\,V_{xf}$

(3) $P_y = (5+3)\,kg\,V_{yf}$

or $V = \sqrt{1.97^2 + 1.42^2}$ = 2.43 m/s NE

$P_{xi} = P_{xf}$: $15.76\,kg\,m/s = 8\,kg \cdot V_{xf}$

(2) w/ units

$V_{xf} = 1.97$ to the right

$P_{yi} = P_{yf}$: $23.7\,kg\,m/s - 12.31\,kg\,m/s = 8\,kg\,V_{yf}$

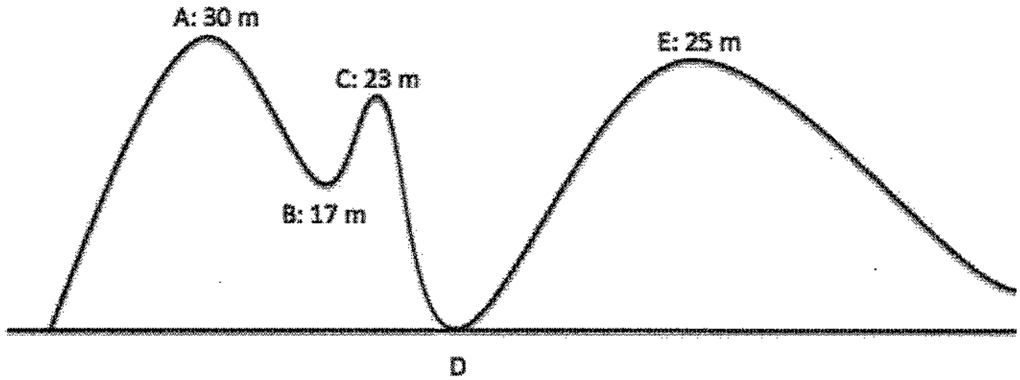
(2) w/ units

$V_{yf} = 1.42$ up

(8)

3. (X pts) You are in charge of designing a roller coaster track, and you've come up with a plan, shown in the image below. The roller coaster car will temporarily come to rest at point A, before plunging through the rest of the ride. The amusement park wants to know how fast the roller coaster car will be going when it hits point D.

Find the velocity of the roller coaster at point D.



Conservation of energy

@ A: E = PE, no KE $\quad E = mgh$ (2)

@ D: E = KE, no PE $\quad E = \frac{1}{2}mv^2$ (2)

$(mgh)_A = \left(\frac{1}{2}mv^2\right)_D$ (2)

Cancel m from both sides

$2g(30m) = V_D^2$

$588 \frac{m^2}{s^2} = V_D^2$

$V_D = 24.25$ m/s (2)

10

4. (X pts) A star has mass m and radius r, and is rotating with a rotational velocity w. It undergoes core collapse, and turns into a neutron star of the same mass but with a much smaller radius.

Which of these statements is true? In addition to choosing a correct response, you must justify with mathematical equations learned in class why the other responses are NOT correct.

I = moment of inertia
L = angular momentum
w = angular velocity

~~a. I remains constant, L increased~~
~~b. L remains constant, $KE_{rotation}$ decreased~~
→ (c.) L remains constant, $KE_{rotation}$ increased
~~d. I decreased, L increased~~
~~e. I decreased, w decreased~~

} 2 for each

– L is always constant, so answers a, d are wrong

– $I = \frac{2}{5}mr^2$, so if r ↓, I ↓.
if $L = I\omega$ is constant, if I ↓ w must ↑.
e is wrong

– $KE = \frac{1}{2}I\omega^2$, or $\frac{1}{2}L\omega$. if L is constant and I ↓, w must ↑, so KE must ↑.
therefore **c is correct** and d is wrong

(6)

5. (X pts) What is the linear speed of a point 5.0 cm from the center of a 55 rpm record?

Change order

$V = r\omega$

$= (5.0 \text{ cm})\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)\left(5.76 \frac{\text{rad}}{\text{sec}}\right)$ (3)

$V = 0.288 \text{ m/s}$

What is the angular speed at this same point?

$\omega = \frac{\Delta\theta}{t} = 55 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$

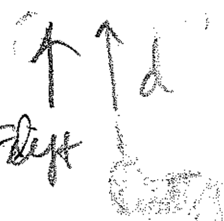
$\omega = 5.76 \frac{\text{rad}}{\text{sec}}$ (3)

(6)

6. (X pts) You pick up a 1.3 kg textbook from the table, raising it to a height of 1 m above the floor. You then hold this book as you walk 5 m across the room.

How much work is done in each stage of the motion?

$W = F d \cos\theta$



a) pick up book. so $\theta = 0°$ (3)

$W = (1.3 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m}) \cos 0°$

$W = +12.74 \text{ J}$

so $\theta = 90°$ (3)

b) carry book

$\cos 90° = 0$, no work is done

(6)

7. (X pts) The Earth has a moment of inertia of 8.008 * 10^{37} kg m^2. If a supervillain wants to destroy the planet, how much work would they need to do in bringing the Earth to rest?

$W = \Delta KE = \frac{1}{2} I \omega^2_{final} - \frac{1}{2} I \omega^2_{initial}$ (2)

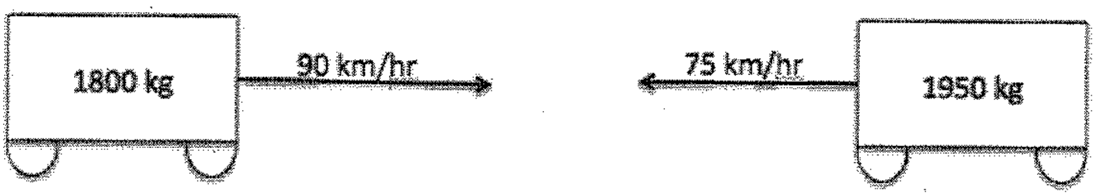
but $\omega_{final} = \emptyset$ b/c stopped

$W = -\frac{1}{2} I \omega^2_{initial} =$ (2)

$-\frac{1}{2}(8.008 \times 10^{37}\ kg\ m^2)(1\ \frac{rev}{day})(\frac{1\ day}{24\ hr})(\frac{1\ hr}{3600\ sec})(\frac{2\pi}{1\ rev})$

$W = -2.11 \times 10^{29}$ J (2)

8. (X pts) Two cars collide in a head-on collision and come to rest, as shown in the image below. How much heat is produced from this collision? (8)



(2) Cons of E says $E_i = E_f$.

(2) $E_i = KE$, E_f = heat

(2) heat E $= E_f = \frac{1}{2}(1800\ kg)(90\ km/hr)^2 + \frac{1}{2}(1950\ kg)(75\ \frac{km}{hr})^2$

heat E $= 12774375\ kg\ (\frac{km}{hr})^2 \left[\frac{1000\ m}{1\ km}\right]^2 \left[\frac{1\ hr}{3600\ sec}\right]^2$

(2) units $= 985{,}677.08\ (kg\ \frac{m^2}{s^2}) \rightarrow J$