PHY2020 - Introduction to Principles of Physics: Exam 3 Practice Problems

Problem 1

Rotational Motion

(a) A wheel rotates around its axle for 2.5 times. How many radians is that?

Solution

$$\theta = 2.5\,\text{rev} \cdot \frac{2\pi\,\text{rad}}{1\,\text{rev}} = 5\pi\,\text{rad} \tag{1}$$

(b) A bicycle with wheels of radius 30 cm is travelling at 2 m/s. How many revolutions per second does each wheel turn through?

Solution

Recall that $\frac{\text{rev}}{\text{s}} = 2\pi\,\frac{\text{rad}}{\text{s}}$ so...

$$v = 2\,\frac{\text{m}}{\text{s}} = R\omega \rightarrow \omega = \frac{2\,\frac{\text{m}}{\text{s}}}{0.3\,\text{m}} = \frac{20}{3}\,\frac{\text{rad}}{\text{s}}\,\frac{\text{rev}}{2\pi\,\text{rad}} = \frac{10}{3\pi}\,\frac{\text{rev}}{\text{s}} \tag{2}$$

(c) A merry-go-round rotates about its center. A person on the outside rim travels with a speed of $v = 5\,\frac{\text{m}}{\text{s}}$.

(i) How fast does someone sitting halfway between the center and the edge travel?

Solution

Since $v = r\omega$ then a person at half the radius, $r = R/2$ has a linear speed half of the speed at the rim where $r = R$.

$$v_{r=R/2} = \frac{v}{2} = 2.5\,\frac{\text{m}}{\text{s}} \tag{3}$$

(ii) If the person on the outside of the merry-go-round has an angular velocity of $\omega = 3\,\frac{\text{rad}}{\text{s}}$, what the angular velocity of the person sitting halfway out have?

Solution

The angular velocity is the same for anyone on merry-go-round. That is to say, every point on the merry-go-round completes one revolution in the same amount of time.

(iii) What is the radius of the merry-go-round?

Solution

$$R = \frac{v}{\omega} = \frac{5\,\frac{\text{m}}{\text{s}}}{3\,\frac{\text{rad}}{\text{s}}} = \frac{5}{3}\,\text{m} \tag{4}$$

Problem 1 continued on next page. . .

(d) A pulley, of mass m, is spinning in a horizontal plane with an angular velocity of ω. You drop a lump of clay of mass m on it, halfway between the axle and the edge of the pulley. What is its angular velocity now?

Solution

Before the lump of clay is dropped onto the pulley, the angular momentum is

$$L_i = I_{\text{disk}}\omega_i = \frac{1}{2}mR^2\omega_i \tag{5}$$

The rotational inertia of the clay lump, halfway out from the axle, is

$$I_{\text{clay}} = m\left(\frac{R}{2}\right)^2 = \frac{1}{4}mR^2 \tag{6}$$

The final angular momentum is then

$$L_f = \left(I_{\text{disk}} + I_{\text{clay}}\right)\omega_f \tag{7}$$

Since angular momentum is conserved $L_f = L_i$, it must be that

$$I_{\text{disk}}\omega_i = \left(I_{\text{disk}} + I_{\text{clay}}\right)\omega_f \rightarrow \omega_f = \frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{clay}}}\omega_i = \frac{1/2}{3/4}\omega_i = \frac{2}{3}\omega_i \tag{8}$$

(e) A meter stick is **balanced** at its midpoint. A 1 kg mass is hanging from the 35 cm mark. At what mark must another 0.5 kg mass be hanging from (so that the meter stick is balanced)?

Solution

If the meter stick is balanced, the **net** torque must be zero (otherwise, the stick would begin to rotate). So, the two masses must produce torques that are equal in magnitude and opposite in sign. Since the meter stick is balanced at its midpoint (0.5 m), the 1 kg mass is to the left of the midpoint so the other mass must be to the right of the midpoint. Thus

$$1\text{ kg}\cdot(0.5 - 0.35)\text{ m} + 0.5\text{ kg}\cdot(0.5 - x)\text{ m} = 0 \rightarrow x = \frac{1\text{ kg}}{0.5\text{ kg}}(0.5 - 0.35)\text{ m} + 0.5\text{ m} = 0.8\text{ m} \tag{9}$$

(f) A 30 cm long wrench is being used to try to turn a nut. There are three forces acting on the handle of the wrench. A 5 N force is applied at right angles to the end of the wrench handle (farthest from the nut). Another 5 N force is applied at right angles to the center of the wrench handle. A 10 N force is applied parallel to the end of the wrench handle. What is the torque on the nut due to each force? (answer for each of the three forces)

Solution

The force parallel to the end of the wrench handle produces no torque. The perpendicular forces produce a net torque of

$$\tau_{\text{net}} = 5\text{N}\cdot\left(0.3 + \frac{0.3}{2}\right)\text{ m} = 2.25\text{ N}\cdot\text{m} \tag{10}$$

Problem 1 continued on next page. . .

(g) A hoop, a solid disk, and a sphere, each with a radius R, begin rolling down the same slope at the same time from the same starting location. In what order will they arrive at the bottom?

Solution

Assuming the same mass M for each object, the object with the smallest rotational inertia will arrive at the bottom first, since, at the bottom, the kinetic energy is the same for each object and

$$KE_{\text{bottom}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v^2 \tag{11}$$

The rotational inertia for each object is

$$I_{\text{hoop}} = MR^2, \qquad I_{\text{disk}} = \frac{1}{2}MR^2, \qquad I_{\text{ball}} = \frac{2}{5}MR^2 \tag{12}$$

thus, the ball arrives first, followed by the disk, followed by the hoop.

(h) A disk and a ball, each with mass M and radius R, have the same angular velocity ω. Calculate the rotational kinetic energy of each.

Solution

The rotational kinetic energy is just

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \tag{13}$$

and so, for the disk

$$KE_{\text{rot}} = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}M\left(R\omega\right)^2 \tag{14}$$

and for the ball

$$KE_{\text{rot}} = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}M\left(R\omega\right)^2 \tag{15}$$

Problem 2

Structure of Matter

(a) Of the three particles that are found in almost all atoms, which ones, if any, are believed to be elementary?

Solution

The electron is believed to be elementary, i.e., not composite.

(b) The nucleus of an atom contains positively charged protons, that electrically repel each other, and electrically neutral neutrons that do not. Since the protons repel each other, what keeps the atomic nucleus together?

Solution

There is a short range force, stronger than the electric force, that is effective only over the distance of about the diameter of a nucleus. It is called the *strong nuclear force* for that reason.

(c) Not long after the formation of our visible universe, there was mostly hydrogen and helium. Where did the other elements that, for example, make up the Earth and our bodies come from?

Solution

The heavier, more complicated elements are formed in the cores of large stars and in the explosion of very large stars.

(d) Which of the common states of matter is typically the least dense? The most dense?

Solution

The solid state is typically more dense (but water is an exception) while the gaseous state is typically the least dense.

(e) Fill in the blanks: The density of substance is   proportional to the mass and   proportional to the volume.

Solution

$$\rho = \frac{M}{V} \tag{16}$$

so

The density of substance is **directly** proportional to the mass and **inversely** proportional to the volume.

(f) Two dogs are identical shapes but different sizes. One is 1 m long and the other is 0.5 m long. The shorter dog has a mass of 4 kg. What is the mass of the longer dog?

Solution

Since volume *scales* as L^3, (remember, the *shapes* remain the same) and we assume the density ρ is the same for each, the mass of the longer dog is

$$M_{\text{longer}} = \left(\frac{L_{\text{longer}}}{L_{\text{shorter}}}\right)^3 M_{\text{shorter}} = \left(\frac{2}{1}\right)^3 M_{\text{shorter}} = 8 \cdot 4\,\text{kg} = 32\,\text{kg} \tag{17}$$

Problem 3

Fluids

(a) A cube of ice floats in water with 10% of the cube above the surface. It is removed from the water and placed in an unknown liquid where it floats with 5% of the cube above the surface. What is the density of the unknown liquid?

Solution

We know, from the first part of the problem, that the density of the ice cube is 90% that of water and, we know from the second part of the problem that the density of the ice cube is 95% that of the unknown liquid thus

$$(0.95)\,\rho_{\text{liquid}} = (0.90)\,\rho_{\text{water}} \rightarrow \rho_{\text{liquid}} = \left(\frac{0.90}{0.95}\right) 1000\,\frac{\text{kg}}{\text{m}^3} = 947\,\frac{\text{kg}}{\text{m}^3} \tag{18}$$

(b) A rock of mass 250 kg and volume $0.1\,\text{m}^3$ is submerged at the bottom of a deep lake. How much upward force is required to lift it off of the bottom of the lake?

Solution

The buoyant force on the rock is

$$F_{\text{buoyant}} = +\rho_{\text{water}} V_{\text{rock}} g \tag{19}$$

The gravitational force on the rock is

$$F_{\text{gravity}} = -M_{\text{rock}} g \tag{20}$$

The force required to lift the rock off of the bottom of the lake is

$$F_{\text{lift}} = F_{\text{gravity}} - F_{\text{buoyant}} = (M_{\text{rock}} - \rho_{\text{water}} V_{\text{rock}})\,g = \left(250\,\text{kg} - 1000\,\frac{\text{kg}}{\text{m}^3}\cdot 0.1\,\text{m}^3\right) 10\,\frac{\text{m}}{\text{s}^2} = 1500\,\text{N} \tag{21}$$

(c) A swimming pool of dimension 5 m(wide) × 50 m(long) × 6 m(deep) has a leak 4 m below the surface. With what pressure does the water squirt out of the leak?

Solution

The pressure is depends on the density of the water and the depth below the surface

$$P_{\text{leak}} = \rho_{\text{water}} h g = 1000\,\frac{\text{kg}}{\text{m}^3}\cdot 4\,\text{m}\cdot 10\,\frac{\text{m}}{\text{s}^2} = 40,000\,\text{Pa} \tag{22}$$

Problem 3 continued on next page. . .

(d) An airplane passenger compartment window has an area of $0.1\ \text{m}^2$. The plane flies to where the air pressure outside is half that at ground level (take the pressure at ground level as 10^5 Pa). If regular atmospheric pressure is maintained in the plane, what is the net force due to air pressure on the window?

Solution

The outward force on the window is

$$F_{\text{window}} = (P_{\text{inside}} - P_{\text{outside}}) \cdot A_{\text{window}} = \frac{1}{2} 10^5\ \text{Pa} \cdot 0.1\ \text{m}^2 = 5 \times 10^3\ \text{N} = 5,000\ \text{N} \tag{23}$$