PHY2020 - Introduction to Principles of Physics: Exam 3

April 8, 2016

Print your name: Solutions

- This exam is closed book, closed notes. A calculator is allowed.
- There are 5 problems. Please read each question carefully.
- Please place a box around your final answers.
- You must show your work in order to get full credit for a correct answer (and don't forget to include units).
- Partial credit will be given for correct intermediate steps even if the final answer is incorrect or the problem is unfinished.

Please read and, by signing below, affirm the UF honor pledge:

"On my honor, I have neither given nor received unauthorized aid in taking this exam."

Signature:

Formula Sheet

Distance travelled (scalar): Δd
Displacement (vector): $\mathbf{d}$
Average speed (scalar): $v = \Delta d/\Delta t$
Average velocity (vector): $\mathbf{v} = \mathbf{d}/\Delta t$
Average acceleration (vector): $\mathbf{a} = \left(\mathbf{v}_f - \mathbf{v}_i\right)/\Delta t$
For constant acceleration: $\mathbf{d} = \mathbf{v}_i \Delta t + \mathbf{a}(\Delta t)^2/2$

Newton's Laws of Motion:
First Law: $\mathbf{F} = 0 \Leftrightarrow \mathbf{v} = \text{constant}$
Second Law: $\mathbf{F} = M\mathbf{a}$
Third Law: **action** = **reaction**

Momentum: $\mathbf{p} = M\mathbf{v}$
Conservation of momentum: **Total final momentum** = **Total initial momentum**

Gravitational Potential Energy: $\text{PE} = Mgh$
Kinetic Energy: $\text{KE} = Mv^2/2$
Work done by a force: $W = |\mathbf{F}| \times \text{distance in direction of force}$
Work-Energy Theorem: $\Delta\text{KE} = W_{\text{net}}$

Uniform circular motion: $a_{\text{cent}} = v^2/R$
Angular displacement: $\theta = \Delta d/R$
Angular velocity: $\omega = v/R$
Rotational Inertia: $I_{\text{hoop}} = MR^2$ $\quad I_{\text{disk}} = MR^2/2$ $\quad I_{\text{ball}} = 2MR^2/5$
Torque: $\tau = F_{\perp}R = I\alpha$
Average angular velocity: $\omega = \Delta\theta/\Delta t$
Average angular acceleration: $\alpha = \Delta\omega/\Delta t$
Angular momentum: $L = I\omega$
Rotational Kinetic Energy: $\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2$

Density: $\rho = \frac{\text{Mass}}{\text{Volume}}$ $\quad$ Pressure: $P = \frac{\text{Force}}{\text{Area}}$
Liquid Pressure: $P = \rho g h$

Problem 1

A disk has a radius of $R = 50\,\text{cm}$, a mass of $M = 2\,\text{kg}$ and a thickness of $4\,\text{cm}$

(a) If the disk rotates through an angle of $\Delta\theta = 4\,\text{rad}$, what distance (in meters) does a point on edge of the disk travel? Is this more than or less than the distance travelled in 1 revolution? (justify)

Solution

$$\Delta d = R\Delta\theta = \frac{1}{2}\,\text{m} \cdot 4\,\text{rad} = 2\,\text{m} \tag{1}$$

One revolution corresponds to $\Delta\theta = 2\pi\,\text{rad} \approx 6.28\,\text{rad} > 4\,\text{rad}$ so the distance travelled is less than the distance travelled in 1 revolution.

(b) If a point halfway from the center of the rotating disk travels a distance of $\Delta d = 1\,\text{m}$ during a time period $\Delta t = \frac{1}{2}\,\text{s}$, what is the average angular velocity of the disk?

Solution

Since the point is *halfway* from the center, it travels *half* the distance of a point on the edge for a given angular displacement: $\Delta d = \frac{1}{2}R\Delta\theta$. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{\Delta d}{R/2}}{\Delta t} = \frac{\frac{1\,\text{m}}{0.25\,\text{m}}}{\frac{1}{2}s} = 8\,\frac{\text{rad}}{\text{s}} \tag{2}$$

(c) If, during this time period, the disk's initial angular velocity is $\omega_i = 6\,\frac{\text{rad}}{\text{s}}$ and the final angular velocity is $\omega_f = 10\,\frac{\text{rad}}{\text{s}}$, what is the disk's average angular acceleration?

Solution

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{10\,\frac{\text{rad}}{\text{s}} - 6\,\frac{\text{rad}}{\text{s}}}{\frac{1}{2}s} = 8\,\frac{\text{rad}}{\text{s}^2} \tag{3}$$

(d) During this time period, what is the net torque acting on the disk?

Solution

We need the rotational inertia of the disk to calculate the torque:

$$I_{\text{disk}} = \frac{1}{2}MR^2 = \frac{1}{2}\cdot 2\,\text{kg}\left(\frac{1}{2}\,\text{m}\right)^2 = \frac{1}{4}\,\text{kg}\cdot\text{m}^2 \tag{4}$$

$$\tau = I_{\text{disk}}\alpha = \frac{1}{4}\,\text{kg}\cdot\text{m}^2 \cdot 8\,\frac{\text{rad}}{\text{s}^2} = 2\,\text{N}\cdot\text{m} \tag{5}$$

Problem 1 continued on next page. . .

(e) What is the net work done on the disk? (Hint: Use the work-energy theorem)

Solution

The work-energy theorem $W_{\text{net}} = \Delta\text{KE}$ tells us that the net work done on the disk equals the change in the disk's kinetic energy. Since this disk only has rotational kinetic energy, we just find the difference of final and initial rotational kinetic energy.

$$\begin{aligned}\Delta\text{KE} &= \left(\frac{1}{2}I_{\text{disk}}\omega_f^2\right) - \left(\frac{1}{2}I_{\text{disk}}\omega_f^2\right) = \frac{1}{2}I_{\text{disk}}\left(\omega_f^2 - \omega_f^2\right) \\ &= \frac{1}{2}\left(\frac{1}{4}\,\text{kg}\cdot\text{m}^2\right)\left(\left(10\,\frac{\text{rad}}{\text{s}}\right)^2 - \left(6\,\frac{\text{rad}}{\text{s}}\right)^2\right) = 8\,\text{J} = W_{\text{net}}\end{aligned} \tag{6}$$

Another approach, which we did not cover in class, is to use $W_{\text{net}} = \tau_{\text{net}}\,\Delta\theta$ (this is analogous to the linear case $W_{\text{net}} = F_{\text{net}}\,\Delta d$.

$$W_{\text{net}} = \tau_{\text{net}}\,\Delta\theta = \tau\left(\omega\cdot\Delta t\right) = \left(2\,\text{N}\cdot\text{m}\right)\left(8\,\frac{\text{rad}}{\text{s}}\cdot\frac{1}{2}s\right) = 8\,\text{J} \tag{7}$$

Problem 2

At the top of a hill with height $h = 10\,\mathrm{m}$, there is a hoop at rest with a radius of $R = 30\,\mathrm{cm}$ and a mass of $M = 500\,\mathrm{g}$. When the hoop reaches the bottom, the hoop continues to roll away on a level surface with a speed v. What is the speed of the hoop? (Hint: write the total kinetic energy as the sum of the ordinary and the rotational kinetic energy).

Solution

We worked this problem in class for both a disk and a hoop. First, we know the kinetic energy at the bottom equals the potential energy at the top:

$$\mathrm{PE}_{\mathrm{top}} = Mgh = \mathrm{KE}_{\mathrm{bot}} \tag{8}$$

The kinetic energy at the bottom is the sum of the ordinary (linear) kinetic energy and the rotational kinetic energy.

$$\mathrm{KE}_{\mathrm{bot}} = \frac{1}{2}Mv^2 + \frac{1}{2}I_{\mathrm{hoop}}\,\omega^2 \tag{9}$$

Since $I_{\mathrm{hoop}} = MR^2$ and $\omega = v/R$, we can write the rotational kinetic energy in terms of the speed v:

$$\frac{1}{2}I_{\mathrm{hoop}}\,\omega^2 = \frac{1}{2}\left(MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 \tag{10}$$

Thus

$$\mathrm{KE}_{\mathrm{bot}} = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2 \tag{11}$$

and so

$$\mathrm{PE}_{\mathrm{top}} = Mgh = Mv^2 = \mathrm{KE}_{\mathrm{bot}} \tag{12}$$

The mass M cancels on both sides and we're left with the remarkably simple expression for the speed:

$$v = \sqrt{gh} = \sqrt{10\,\frac{\mathrm{m}}{\mathrm{s}^2}\cdot 10\,\mathrm{m}} = \sqrt{100\,\frac{\mathrm{m}^2}{\mathrm{s}^2}} = 10\,\frac{\mathrm{m}}{\mathrm{s}} \tag{13}$$

Problem 3

(a) What is the density of the disk in problem 1?

Solution

The volume of a disk is the circular area $A = \pi R^2$ of the disk multiplied by the thickness d of the disk:

$$V_{\text{disk}} = \pi R^2 \cdot d = \pi \left(\frac{1}{2}\,\text{m}\right)^2 \cdot 0.04\,\text{m} = 0.01\,\pi\,\text{m}^3 \tag{14}$$

$$\rho = \frac{M}{V_{\text{disk}}} = \frac{2\,\text{kg}}{0.01\,\pi\,\text{m}^3} = \frac{200}{\pi}\,\frac{\text{kg}}{\text{m}^3} \tag{15}$$

(b) If the disk in problem 1 is scaled up by a factor of 3 (assume it is made of the same substance), what is the mass of the scaled up disk?

Solution

We know from the scaling laws that volume scales as the *cube* of the scale factor

$$V' = \left(3^3\right) V = 27 \cdot V \tag{16}$$

and thus, so does the mass

$$M' = \rho V' = \rho\,(27 \cdot V) = 27 \cdot M = 27 \cdot 2\,\text{kg} = 54\,\text{kg} \tag{17}$$

(c) What is the rotational inertia of the scaled up disk?

Solution

The radius of the scaled up disk is three times larger than the original disk: $R' = 3R = 1.5\,\text{m}$.

$$I' = \frac{1}{2} M' R'^2 = \frac{1}{2}\,(54\,\text{kg})\,(1.5\,\text{m})^2 = 60.75\,\text{kg} \cdot \text{m}^2 \tag{18}$$

(d) If the disk in problem 1 requires N cans of paint to finish, how many cans of paint are required to finish both disks?

Solution

We know from the scaling laws that area scales as the *square* of the scale factor thus

$$A' = \left(3^2\right) V = 9 \cdot A \tag{19}$$

Since the amount of paint is proportional to the area, the scaled up disk requires 9N cans of paint. So, the number of cans required to finish both disks is $9N + N = 10N$ cans of paint.

Problem 4

A long tube, narrow at one end and wide at the other end, is filled with a liquid and sealed by pistons at each end. The cross sectional area of the piston in the narrow end is $A_1 = 100\,\text{cm}^2$. A force of $F_1 = 100\,\text{N}$ pushes inward on this piston. Neither piston is moving.

(a) What is the pressure within the liquid?

Solution

The pressure in the liquid, due to the force F_1, is transmitted uniformly throughout the liquid, according to Pascal's Principle, and is equal to

$$P = \frac{F_1}{A_1} = \frac{100\,\text{N}}{100\,\text{cm}^2} \cdot \frac{1\,\text{cm}^2}{(0.01\,\text{m})^2} = \frac{100\,\text{N}}{0.01\,\text{m}^2} = 10\,\text{kPa} \tag{20}$$

(b) A force meter attached to the piston at the wide end measures an outward force of $F_2 = 100\,\text{kN}$. What is the cross sectional area of this piston?

Solution

We're given F_2 and we've calculate the pressure P so

$$A_2 = \frac{F_2}{P} = F_2 \frac{A_1}{F_1} = 10^5\,\text{N} \cdot \frac{0.01\,\text{m}^2}{10^2\,\text{N}} \cdot = 10\,\text{m}^2 \tag{21}$$

(c) If the wide end piston is allowed to move outward a distance of 1 mm with F_2 remaining constant, how far must the narrow end piston have moved? (Hint: consider the work done by each force).

Solution

The work done by F_1 must equal the work done by F_2 thus

$$F_1 \cdot \Delta d_1 = F_2 \Delta d_2 \Rightarrow \Delta d_1 = \frac{F_2}{F_1}\,\Delta d_2 = \frac{10^5\,\text{N}}{10^2\,\text{N}} \cdot 0.001\,\text{m} = 1\,\text{m} \tag{22}$$

Problem 5

A cubicle box of side length $L = 1$ m is half filled with water.

(a) What is the mass of the water within the box?

Solution

If the box were *filled* with water, the mass of the water would be

$$M_{\text{water}} = \rho_{\text{water}} V_{\text{box}} = 1000\,\frac{\text{kg}}{\text{m}^3} \cdot 1\,\text{m}^3 = 1000\,\text{kg} \tag{23}$$

However, the box is only *half filled* with water and so

$$M_{\text{water}} = \frac{1}{2} \cdot 1000\,\text{kg} = 500\,\text{kg} \tag{24}$$

(b) There is a small hole in the side of the box through which water squirts out with a pressure of 2×10^3 Pa. What is the height of the hole *from the bottom of the box*?

Solution

The pressure at a distance h, *below the surface of the water*, is

$$P = \rho g h = 2\,\text{kPa} \Rightarrow h = \frac{2\,\text{kPa}}{10\,\frac{\text{m}}{\text{s}^2} \cdot 1000\,\frac{\text{kg}}{\text{m}^3}} = 0.2\,\text{m} \tag{25}$$

But this isn't what was asked for. The hole is 0.2 m below the surface of the water but, because the box is only half filled with water, the surface of the water is $\frac{1}{2}$ m above the bottom of the box. Thus, the height of the hole *above the bottom of the box* is $0.5\,\text{m} - 0.2\,\text{m} = 0.3\,\text{m}$

(c) Water is added to the box until the pressure at the hole doubles. What is the mass of the water in the box now?

Solution

For twice the pressure at the hole, the water level must increase until the hole is twice the distance below the surface of the water. Thus, water must be added until the surface of the water is $2 \cdot 0.2\,\text{m} = 0.4\,\text{m}$ above the hole or 0.7 m above the bottom of the box. The box is now 7/10 filled with water and so, the mass of the water in the box is 700 kg.