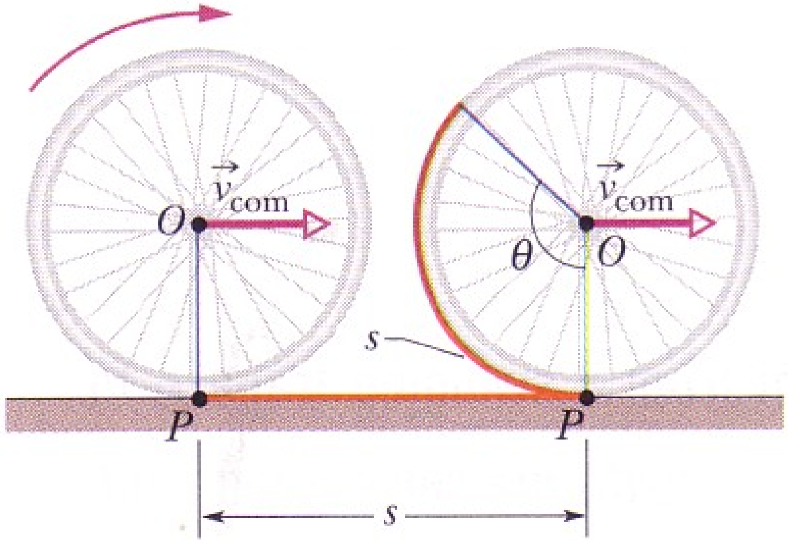
Class 27 - Rolling, Torque and Angular Momentum
Chapter 11 - Wednesday October 27th

- Rolling motion as translation plus rotation
- Rolling motion as pure rotation
- The kinetic energy of rolling
- The forces of rolling motion
- Examples of rolling motion

Reading: pages 275 thru 281 (chapter 11) in HRW
Read and understand the sample problems
Assigned problems from chapter 11:
2, 6, 8, 12, 22, 24, 32, 38, 40, 50, 54, 64

Rolling motion as rotation and translation

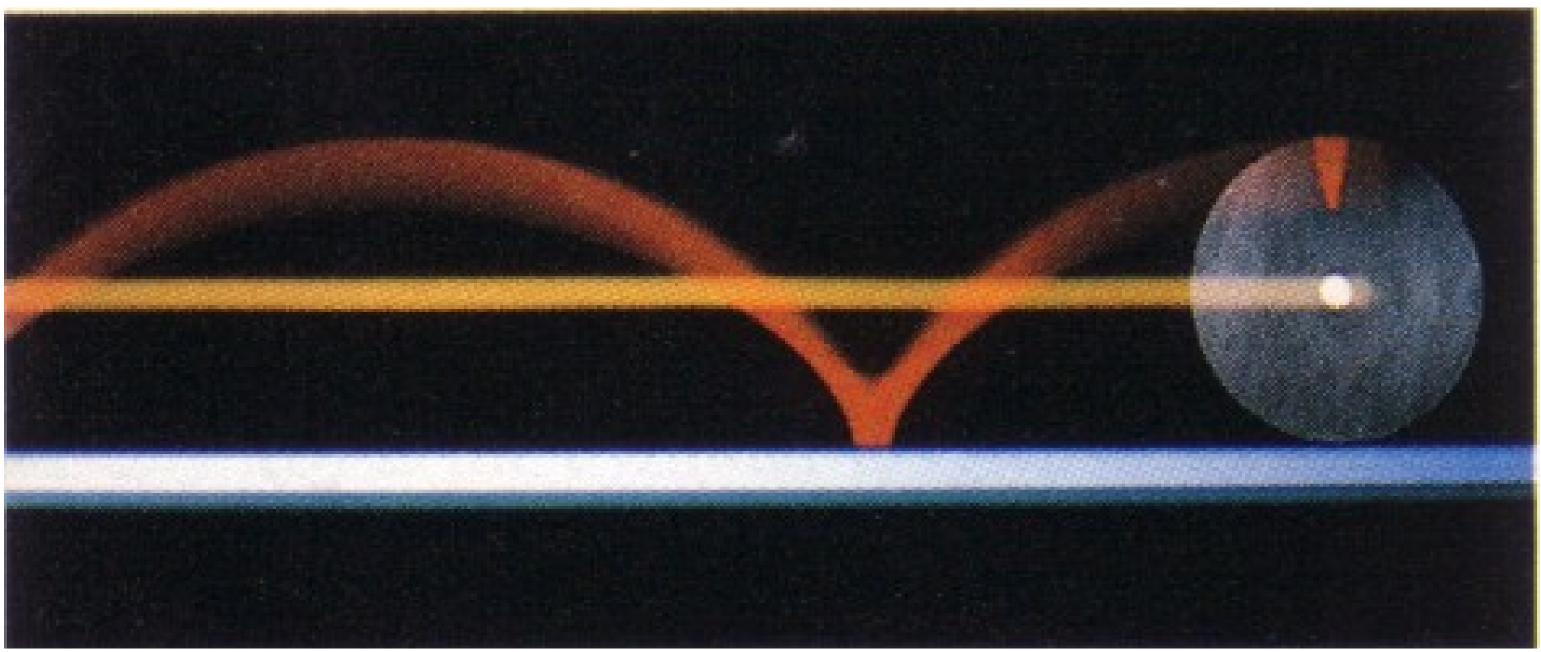


$$s = \theta R$$

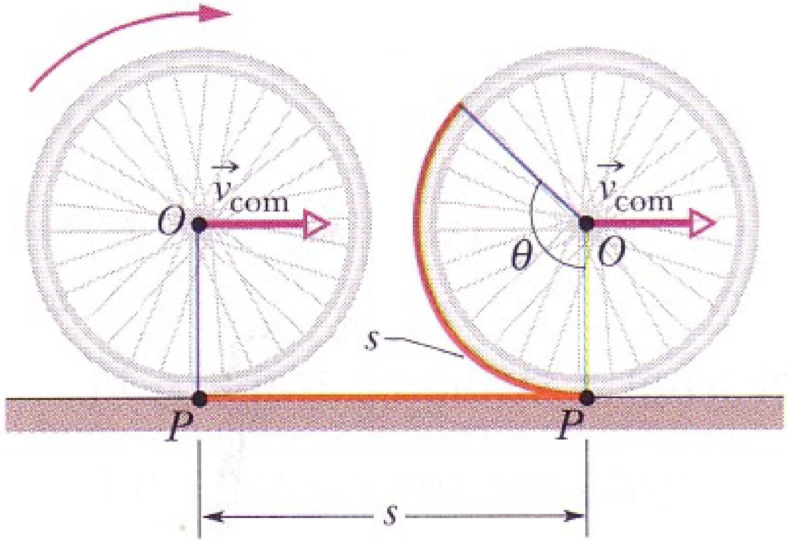
The wheel moves with speed *ds/dt*

$$\Rightarrow v_{com} = \omega R$$

Another way to visualize the motion:



Rolling motion as rotation and translation



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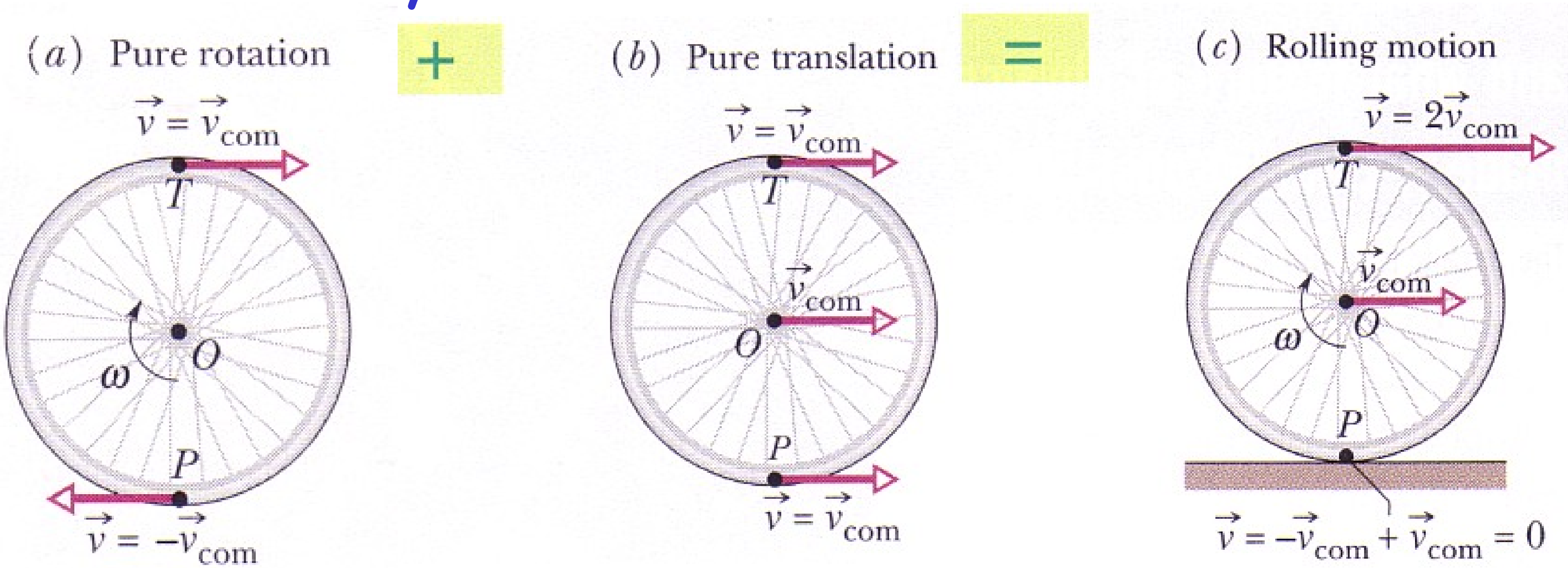
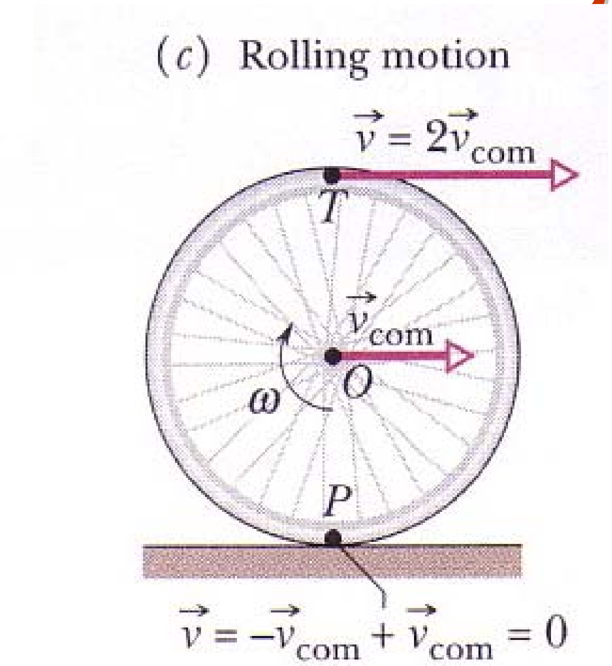
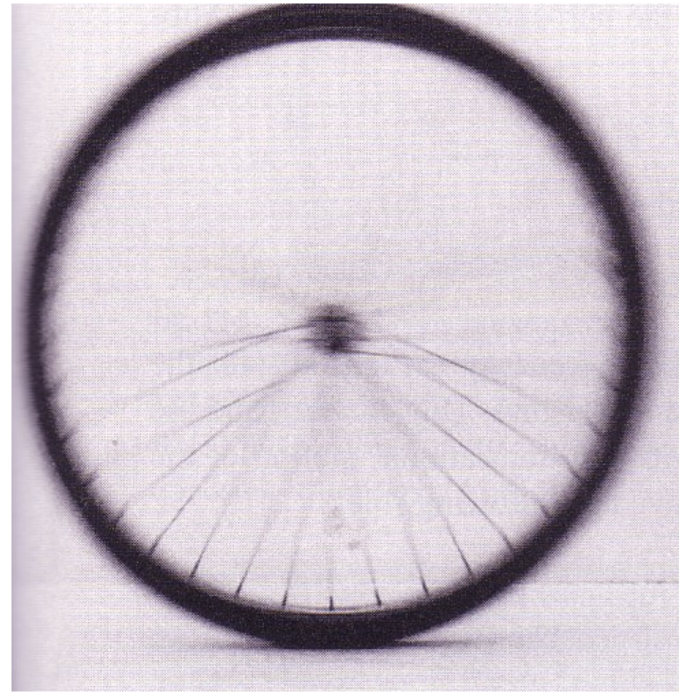


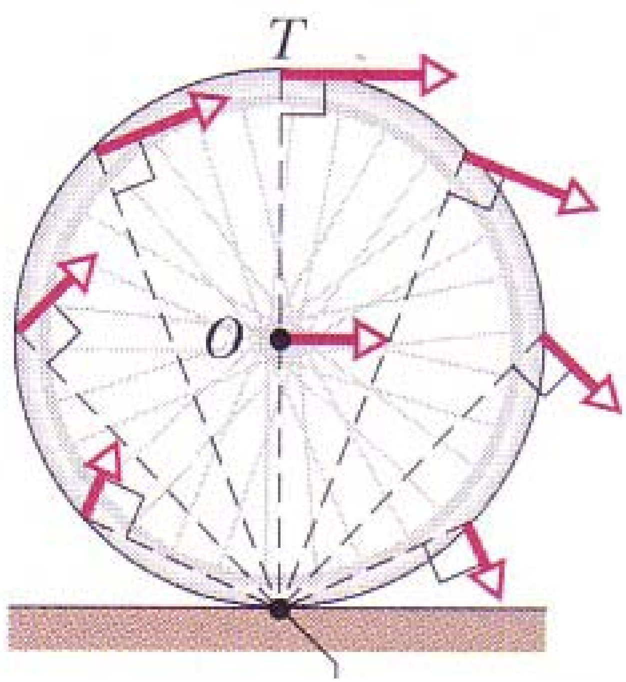
Rolling motion as pure rotation





$$v_{top} = (\omega)(2R)$$

$$= 2(\omega R) = 2v_{com}$$



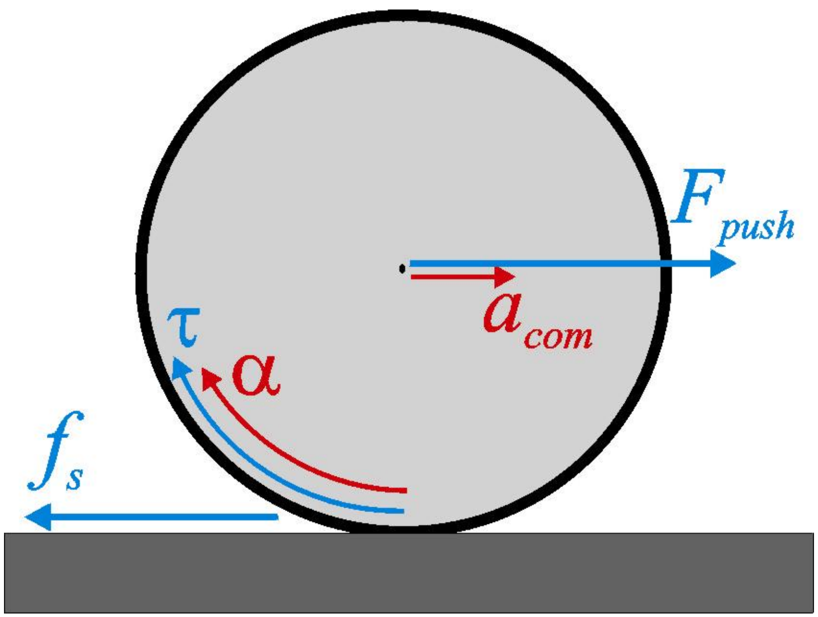
The kinetic energy of rolling

$$K = \tfrac{1}{2} I_P \omega^2 \qquad I_P = I_{com} + MR^2$$

$$K = \tfrac{1}{2} I_{com} \omega^2 + \tfrac{1}{2} MR^2 \omega^2$$

$$K = \tfrac{1}{2} I_{com} \omega^2 + \tfrac{1}{2} M v_{com}^2 = K_r + K_t$$

The forces of rolling (role of friction)



•If the *c.o.m.* moves with constant velocity, and the wheel rotates with constant angular speed $\omega = v_{com}/R$, then there are no net forces on the wheel.

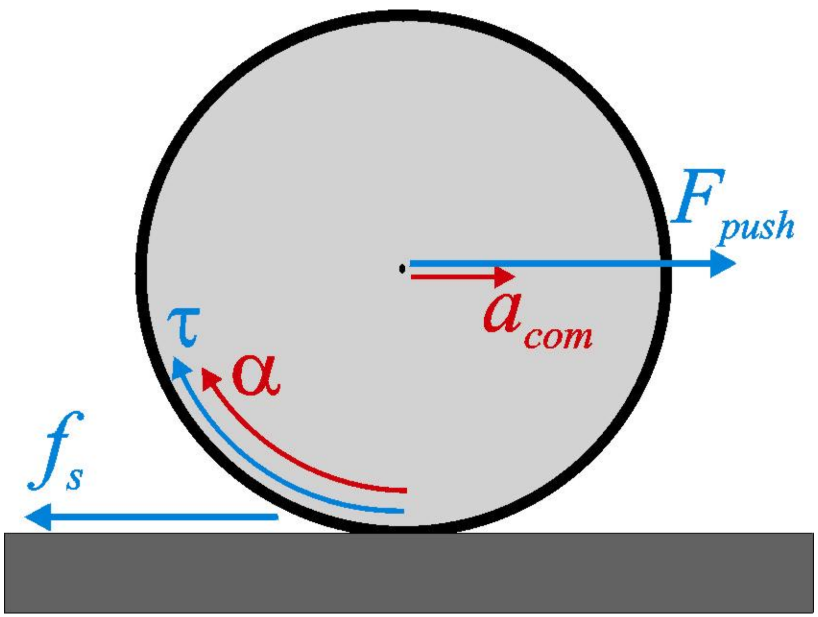
•If however, the wheel has an angular acceleration, then there must be a frictional force at P in order for the wheel to move.

•If the driving mechanism is a linear force, e.g. someone pushing the wheel through its center of mass, then the frictional force opposes the pushing force.

•The sum of the two forces provide the linear acceleration.

•Meanwhile, the friction force has a moment arm about the *c.o.m.*, i.e. it provides a torque which, in part, causes the angular acceleration.

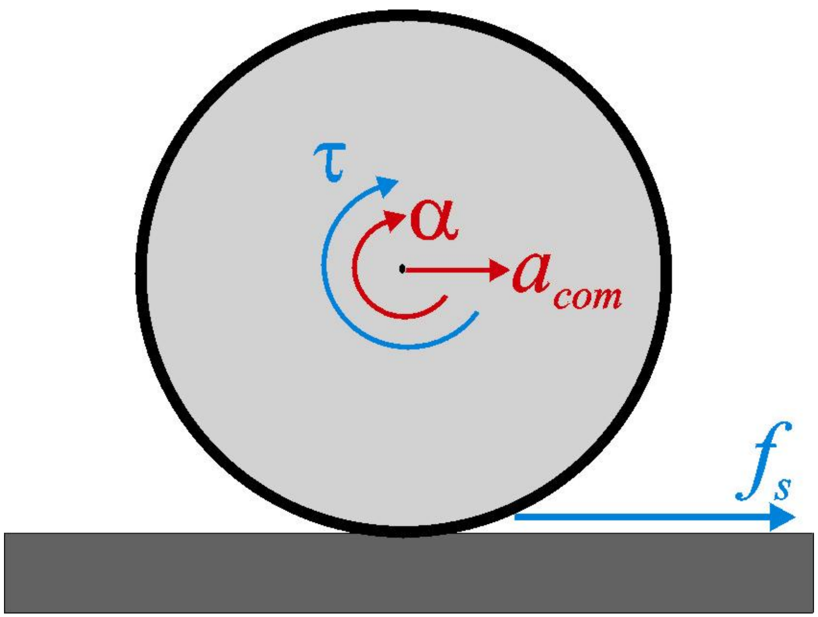
The forces of rolling (role of friction)



- If the *c.o.m.* moves with constant velocity, and the wheel rotates with constant angular speed $\omega = v_{com}/R$, then there are no net forces on the wheel.
- If however, the wheel has an angular acceleration, then there must be a frictional force at P in order for the wheel to move.

- If the driving mechanism is a linear force, e.g. someone pushing the wheel through its center of mass, then the frictional force opposes the pushing force.
- The sum of the two forces provide the linear acceleration.
- One can also attribute the angular acceleration to the fact that F_{push} has a moment arm about the point where the wheel makes contact with the road.

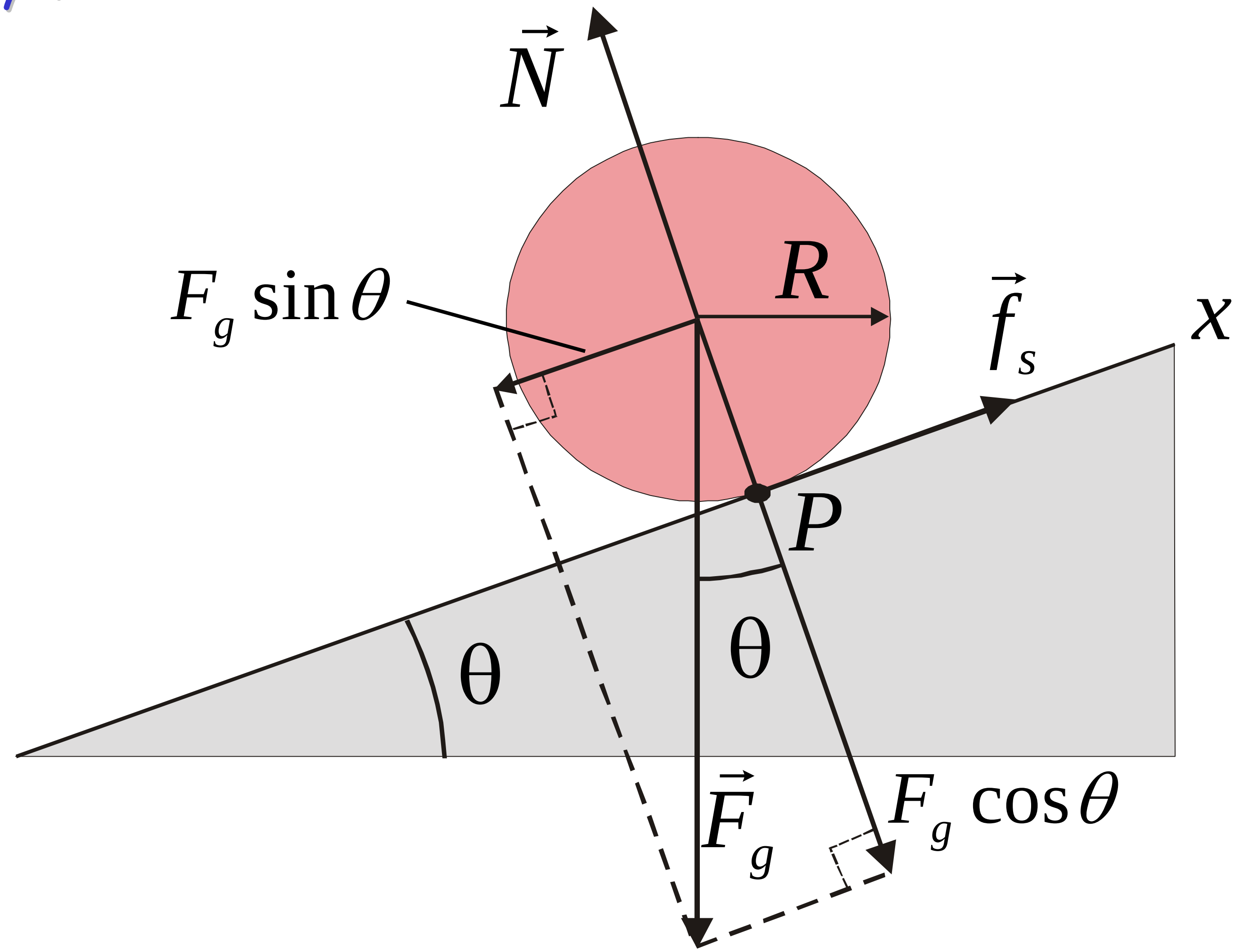
The forces of rolling (role of friction)



- If the *c.o.m.* moves with constant velocity, and the wheel rotates with constant angular speed $\omega = v_{com}/R$, then there are no net forces on the wheel.
- If however, the wheel has an angular acceleration, then there must be a frictional force at P in order for the wheel to move.
- If, on the other hand, the driving mechanism is rotational, *i.e.* due to the torque of a motor, then the frictional force is in the opposite direction, and *it* causes the linear acceleration.

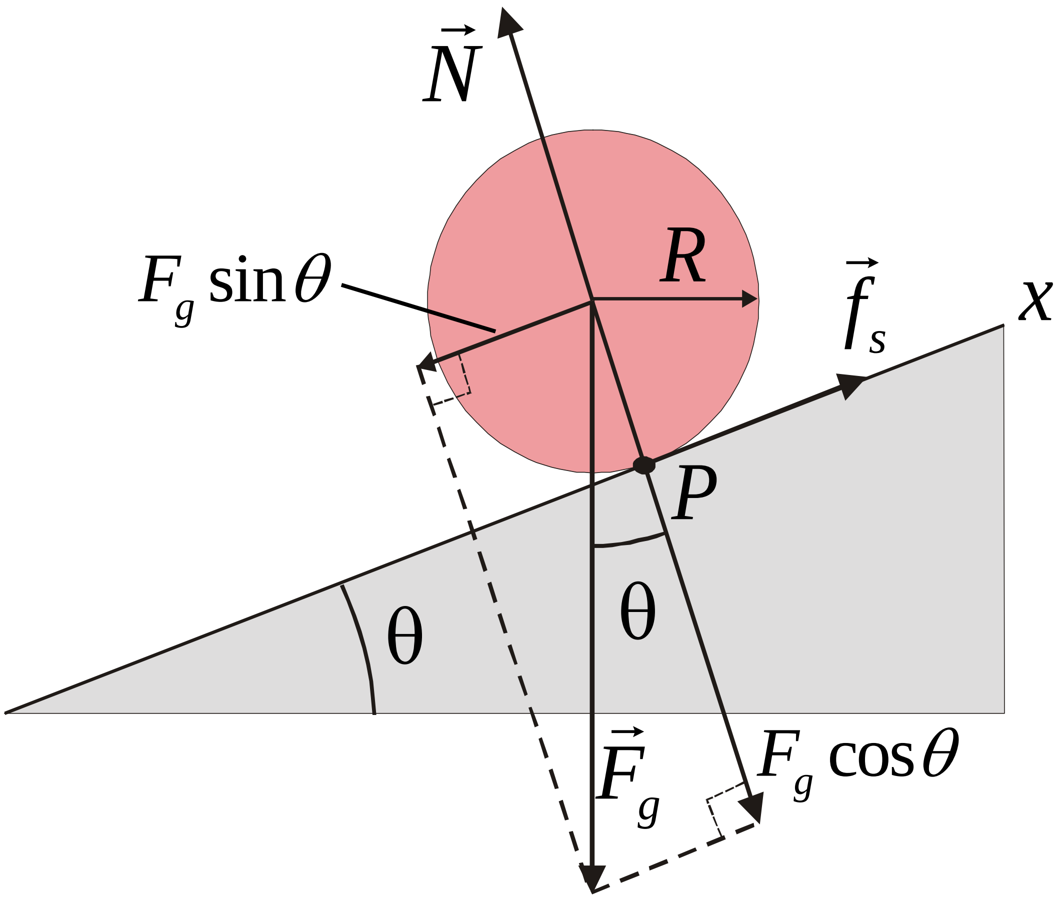
Rolling down a ramp

- The frictional force is essential for the rolling motion.
- If one analyzes the motion about the center of the disk, f_s is the only force with a moment arm.



Rolling down a ramp

- However, we do not really have to compute f_s (see section 12-3).
- We can, instead, analyze the motion about P, in which case, $F_g\sin\theta$ is the only force component with a moment arm about P.



Use: torque = $I\alpha$

$$R \times F_g \sin\theta = I_P \alpha$$

$$a_{com} = -\alpha R$$

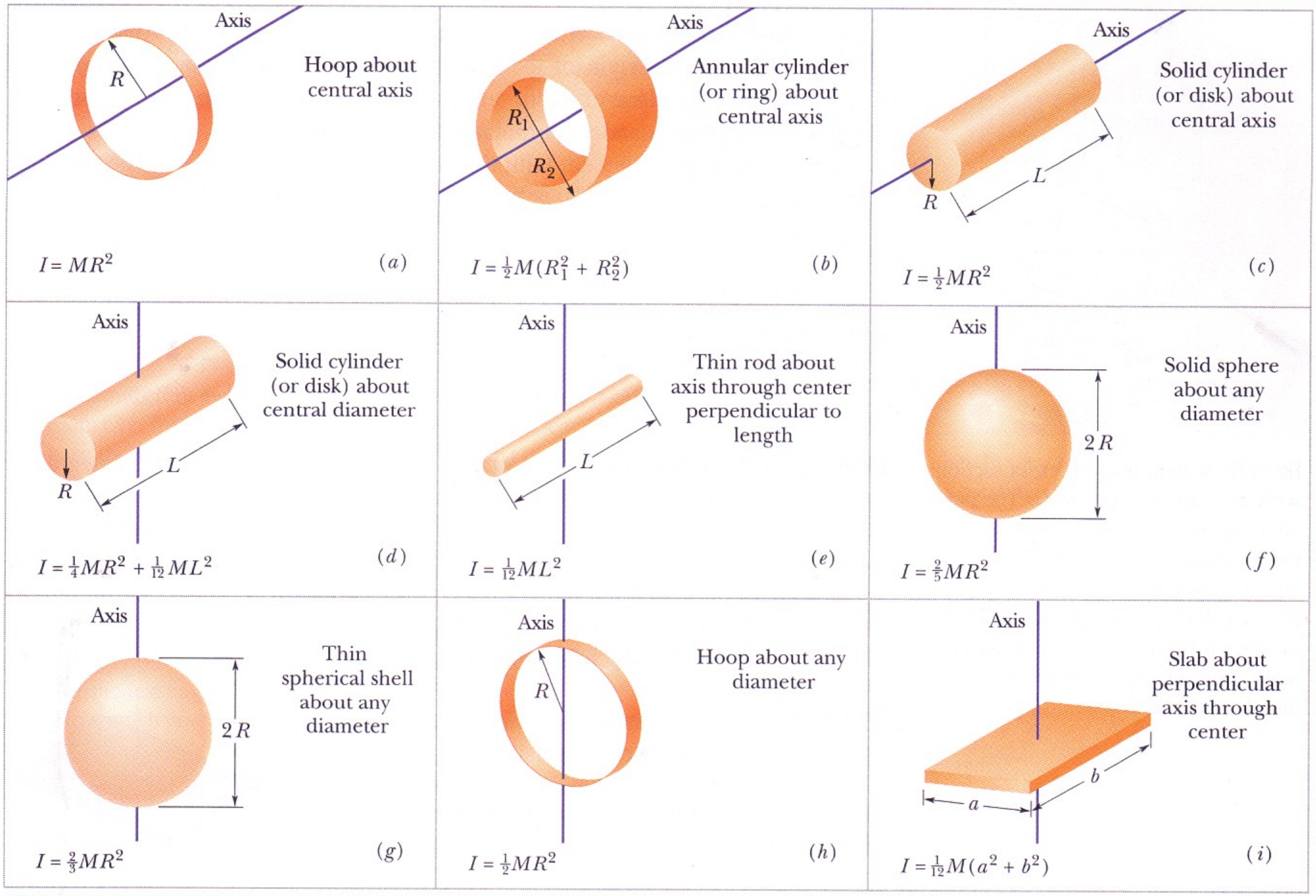
Thus:

$$MR^2 g \sin\theta = -I_P a_{com}$$

$$I_P = I_{com} + MR^2$$

$$a_{com} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$$

Some rotational inertia

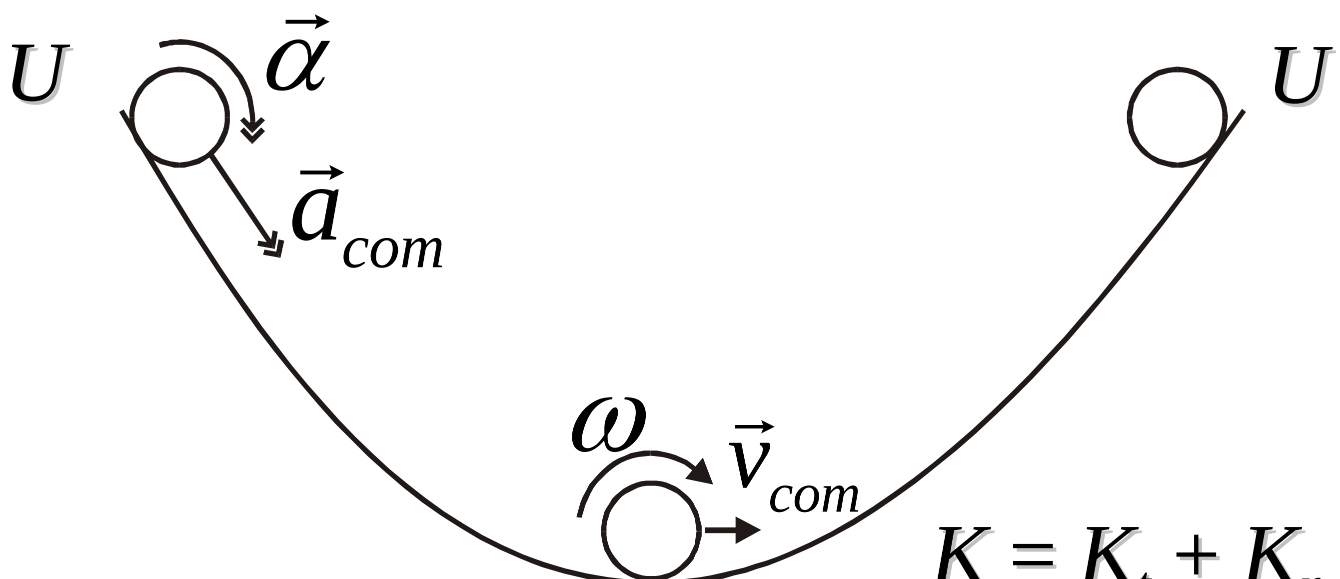


More on rolling

$$a_{com} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$$

This is the same as for the frictionless incline ($a_{com} = g\sin\theta$) with the additional I_{com}/MR^2 term in the denominator.

- Mechanical energy is NOT lost as a result of frictional forces.
- It is, instead, converted to rotational kinetic energy.
- This rotational kinetic is mechanical, so it may be converted back to translational kinetic energy, or gravitational potential energy.

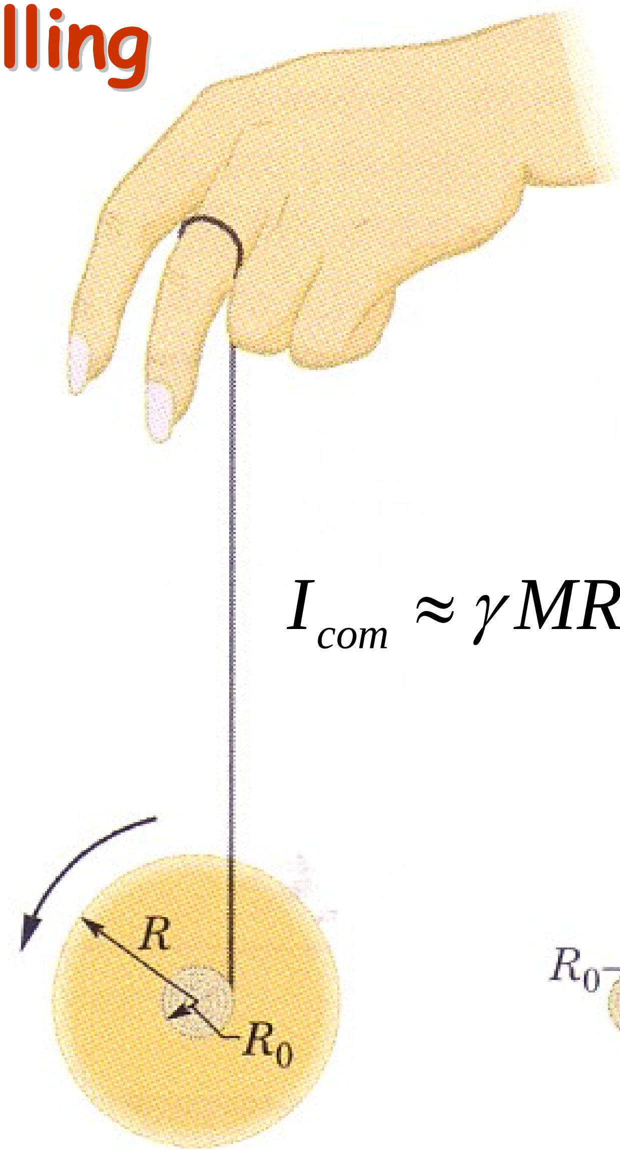


$$K = K_t + K_r$$

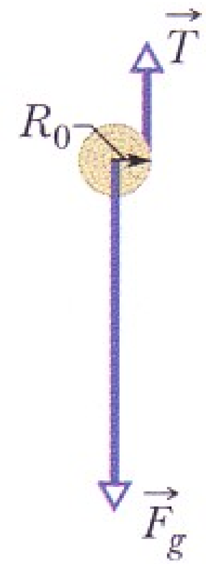
More on rolling

$$a_{com} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$$

- The Yo-yo is essentially the same as the disc rolling down the incline, except that the string plays the role of the slope and the tension in the string plays the role of the friction.
- Thus, $\theta = 90^o$, and the relevant radius is R_o.
- However, one should use R when calculating I_{com}.



$$I_{com} \approx \gamma MR^2$$



$$\Rightarrow \qquad a_{com} = -\frac{g}{1 + I_{com}/MR_o^2} = -\frac{g}{1 + \gamma R^2/R_o^2}$$