- **Exam 3: Friday December 3rd, 8:20pm to 10:20pm**
- **You must go to the following locations based on the 1st letter of your last name:**

A to F	WEIL270 (Weil Hall)
G to M	**WM 100 (Williamson Hall)**
N to Z	FAB103/105 (Fine Arts B)

- **Review sessions: Tuesday Nov. 30 (Hill) and Thurs. Dec. 2 (Woodard), 6:15 to 8:10pm in NPB 1001 (HERE!)**

- **Final Exam (cumulative): Tuesday December 14th, 12:30pm to 2:30pm.**
- **Room assignments: A to K in NPB1001 (in here); L to Z in Norman Hall 137.**
- **Two more review sessions: Dec. 7 and Dec. 9, 6:15 to 8:10pm in NPB1001 (HERE!)**

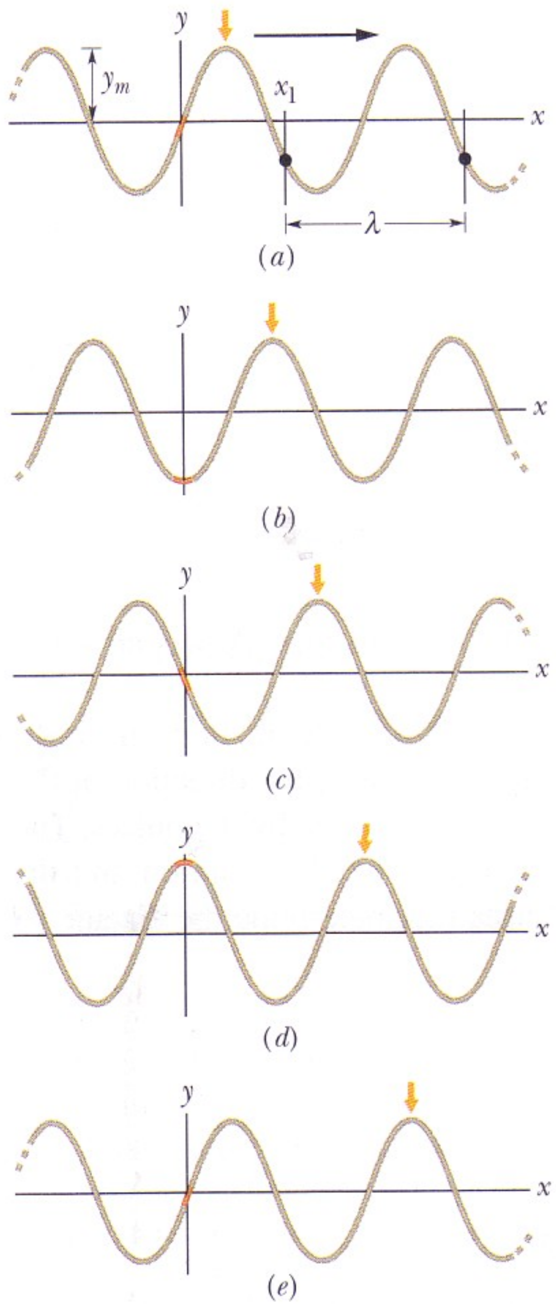
Class 38 - Waves I
Chapter 16 - Monday November 29th

- QUICK review of traveling waves
- Wave speed and the wave equation
- HiTT problem(s)
- Energy in traveling waves
- Introduction to wave interference

Reading: pages 413 to 437 (chapter 16) in HRW
Read and understand the sample problems
Assigned problems from chapter 16 (due Dec. 2nd!):
6, 20, 22, 24, 30, 34, 42, 44, 66, 70, 78, 82

Review - wavelength and frequency

Transverse wave



Amplitude

Phase

Displacement

$$y(x,t) = y_m \sin(\overbrace{kx \mp \omega t + \phi}^{\text{Phase}})$$

angular wavenumber

angular frequency

Phase shift

$$k = \frac{2\pi}{\lambda}$$

k is the angular wavenumber.

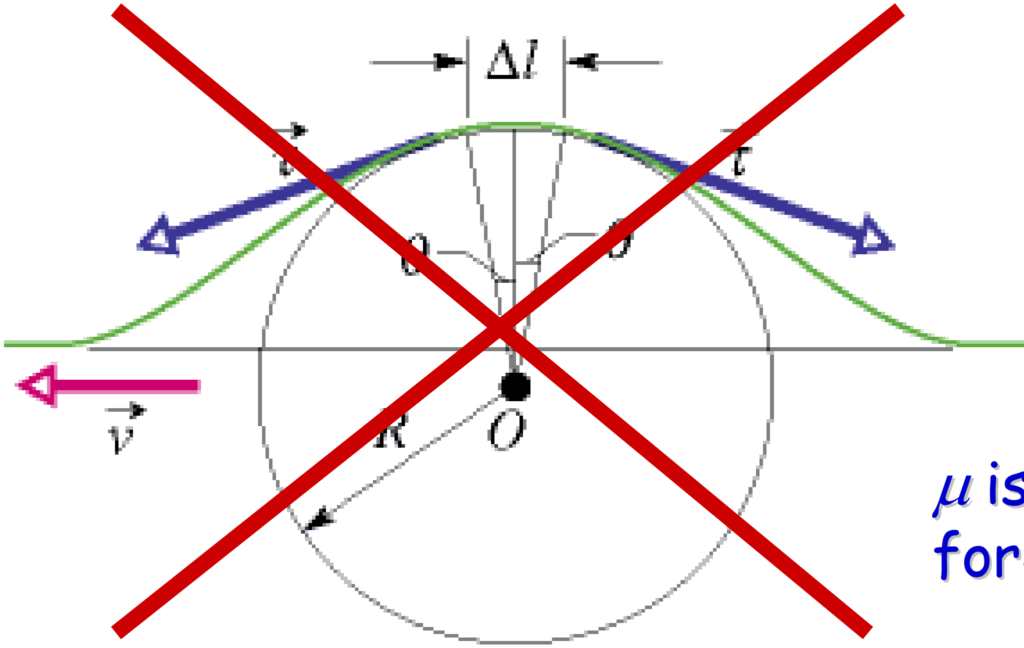
$$\omega = \frac{2\pi}{T}$$

ω is the angular frequency.

frequency $$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

velocity $$v = \pm\frac{\omega}{k} = \pm\frac{\lambda}{T} = \pm f\lambda$$

Traveling waves on a stretched string



Dimensional analysis

$$\Delta m = \mu \Delta l$$

μ is the string's linear density, or force per unit length.

- Tension τ provides the restoring force (kg.m.s^{-2}) in the string. Without tension, the wave could not propagate.
- The mass per unit length μ (kg.m^{-1}) determines the response of the string to the restoring force (tension), through Newtorn's 2nd law.
- Look for combinations of τ and μ that give dimensions of speed (m.s^{-1}).

$$v = C\sqrt{\frac{\tau}{\mu}}$$

The wave equation

$$\frac{\tau}{\mu}\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

Derivation on page 425 of HRW
Or in class notes from Nov. 24th

- General solution:

$$y(x,t) = y_m \sin\left(kx \pm \omega t\right) \quad \text{or} \quad y(x,t) = y_m f\left(kx \pm \omega t\right)$$

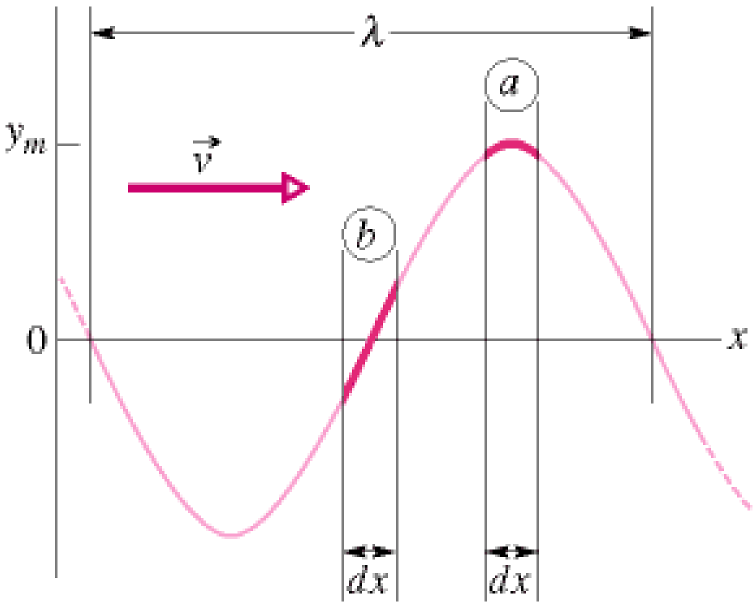
$$\frac{\partial^2 y}{\partial x^2} = -k^2 y\left(x,t\right) \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 y\left(x,t\right)$$

$$\Rightarrow \qquad -\frac{\tau}{\mu}k^2 = -\omega^2 \quad \text{or} \quad \frac{\omega^2}{k^2} = v^2 = \frac{\tau}{\mu}$$

$$\text{i.e.} \qquad v = \sqrt{\frac{\tau}{\mu}}$$

Energy in traveling waves

$$y(x,t) = y_m \sin\left(kx - \omega t\right)$$



$$dm = \mu dx$$

Kinetic energy: $dK = {}^{1}/_{2}\, dm\, v_y^{\,2}$

$$v_y = \frac{\partial y}{\partial t} = -\omega y_m \cos\left(kx - \omega t\right)$$

$$dK = \tfrac{1}{2}\left(\mu dx\right)\left(-\omega y_m\right)^2 \cos^2(kx - \omega t)$$

Divide both sides by dt, where $dx/dt = v_x$

$$\frac{dK}{dt} = \tfrac{1}{2}\mu v_x \omega^2 y_m^2 \cos^2(kx - \omega t)$$

Similar expression for elastic potential energy

$$\frac{dU}{dt} = \tfrac{1}{2}\mu v_x \omega^2 y_m^2 \cos^2(kx - \omega t)$$

$$\boxed{P_{avg} = 2\times\tfrac{1}{2}\mu v \omega^2 y_m^2 \left\langle \cos^2(kx - \omega t)\right\rangle = 2\times\tfrac{1}{2}\mu v\omega^2 y_m^2 \times \tfrac{1}{2} = \tfrac{1}{2}\mu v \omega^2 y_m^2}$$

Energy is pumped in an oscillatory fashion down the string

Note: I dropped the subscript on v since it represents the wave speed

The principle of superposition for waves

- It often happens that waves travel simultaneously through the same region, *e.g.*
  - Radio waves from many broadcasters
  - Sound waves from many musical instruments
  - Different colored light from many locations from your TV
- Nature is such that all of these waves can exist without altering each others' motion
- Their effects simply add
- This is a result of the principle of superposition, which applies to all harmonic waves, *i.e.* waves that obey the linear wave equation

$$v^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

- And have solutions: $y(x,t) = y_m f\left(kx \pm \omega t\right) \quad \text{or} \quad y_m \sin\left(kx \pm \omega t\right)$

The principle of superposition for waves

- If two waves travel simultaneously along the same stretched string, the resultant displacement y' of the string is simply given by the summation

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

where y_1 and y_2 would have been the displacements had the waves traveled alone.

- This is the principle of superposition.

Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

Overlapping waves do not in any way alter the travel of each other

Link 3

Interference of waves

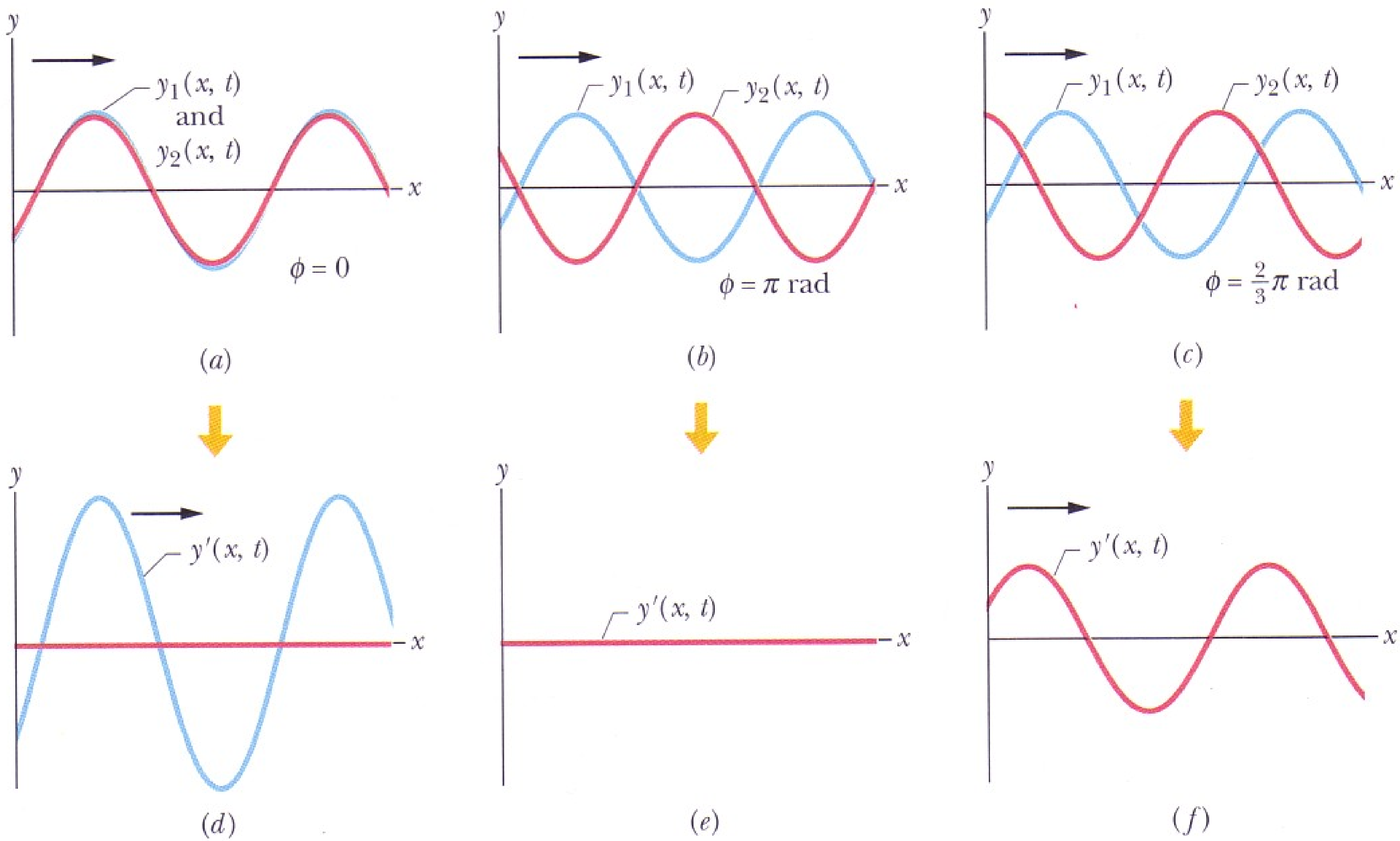
- Suppose two sinusoidal waves with the same frequency and amplitude travel in the same direction along a string, such that

$$y_1 = y_m \sin\left(kx - \omega t\right)$$

$$y_2 = y_m \sin\left(kx - \omega t + \phi\right)$$

- The waves will add.
- If they are in phase (*i.e.* $\phi = 0$), they combine to double the displacement of either wave acting alone.
- If they are out of phase (*i.e.* $\phi = \pi$), they combine to cancel everywhere, since $\sin(\alpha) = -\sin(\alpha + \pi)$.
- This phenomenon is called interference.

Interference of waves



Interference of waves

- Mathematical proof:

$$y_1 = y_m \sin(kx - \omega t)$$

$$y_2 = y_m \sin(kx - \omega t + \phi)$$

Then:

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

$$= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

But:

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)$$

Phase shift

So:

$$y'(x,t) = \underbrace{\left[2y_m \cos\tfrac{1}{2}\phi\right]}_{\textbf{Amplitude}}\underbrace{\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)}_{\textbf{Wave part}}$$

Interference of waves

$$y'(x,t) = \left[2y_m \cos\tfrac{1}{2}\phi\right]\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)$$

If two sinusoidal waves of the same amplitude and frequency travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in the same direction.

- If $\phi = 0$, the waves interfere constructively, $\cos\frac{1}{2}\phi = 1$ and the wave amplitude is $2y_m$.
- If $\phi = \pi$, the waves interfere destructively, $\cos(\pi/2) = 0$ and the wave amplitude is 0, *i.e.* no wave at all.
- All other cases are intermediate between an amplitude of 0 and $2y_m$.
- Note that the phase of the resultant wave also depends on the phase difference.