Class 9 - Review of motion in 2D and 3D
Chapter 4 - Monday September 13th

- Exam instructions
- Brief review
- Example problems

Reading: pages 58 thru 75 (chapter 4) in HRW
Read and understand the sample problems

- **Exam tonight: 8:20-10:20pm**

1st letter of last name	Room assignment
A to F	FLI 50 (Flint)
G to O	CSE A101 (Comp. Sci. & Eng.)
P to Z	FAB 103 or 105 (fine arts bldg)

WebAssign is NO LONGER FREE!! You must register

Two-dimensional kinematics

Position: $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$

Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}$

Velocity: $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\hat{\mathrm{i}} + \dfrac{dy}{dt}\hat{\mathrm{j}} + \dfrac{dz}{dt}\hat{\mathrm{k}}$

Acceleration: $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{dv_x}{dt}\hat{\mathrm{i}} + \dfrac{dv_y}{dt}\hat{\mathrm{j}} + \dfrac{dv_z}{dt}\hat{\mathrm{k}}$

Projectile motion

$$x - x_0 = (v_0 \cos\theta_0)t \qquad 4-21$$

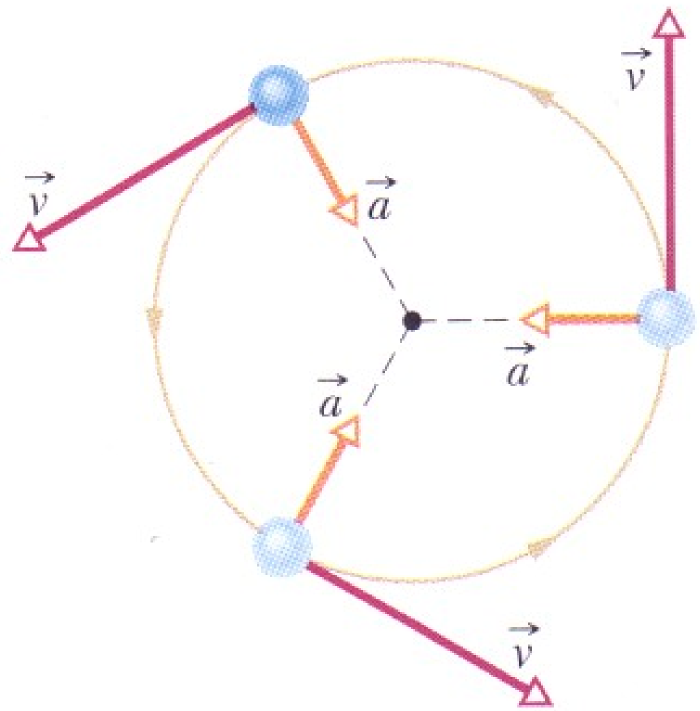
$$v_x = v_0 \cos\theta_0$$

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2 \qquad 4-22$$

$$v_y = v_0 \sin\theta_0 - gt \qquad 4-23$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0) \qquad 4-24$$

Uniform circular motion



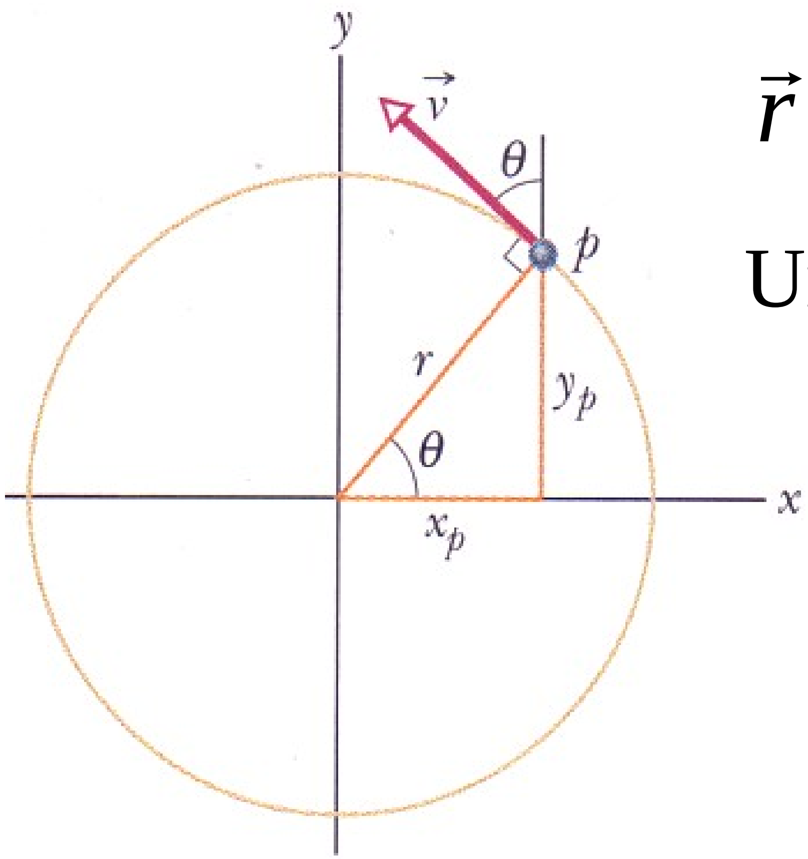
- Although v does not change, the direction of the motion does, *i.e.* the velocity (a vector) changes.
- Thus, there is an acceleration associated with the motion.
- We call this a centripetal acceleration.

Acceleration: $a = \dfrac{v^2}{r}$ Period: $T = \dfrac{2\pi r}{v}$

Frequency: $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\dfrac{v}{r}; \;\; \omega = 2\pi f = \dfrac{v}{r}$

- Since v does not change, the acceleration must be perpendicular to the velocity.

Analyzing the motion



$$x_p = r\cos\theta \qquad y_p = r\sin\theta$$

$$\vec{r} = x_p\hat{\mathrm{i}} + y_p\hat{\mathrm{j}}$$

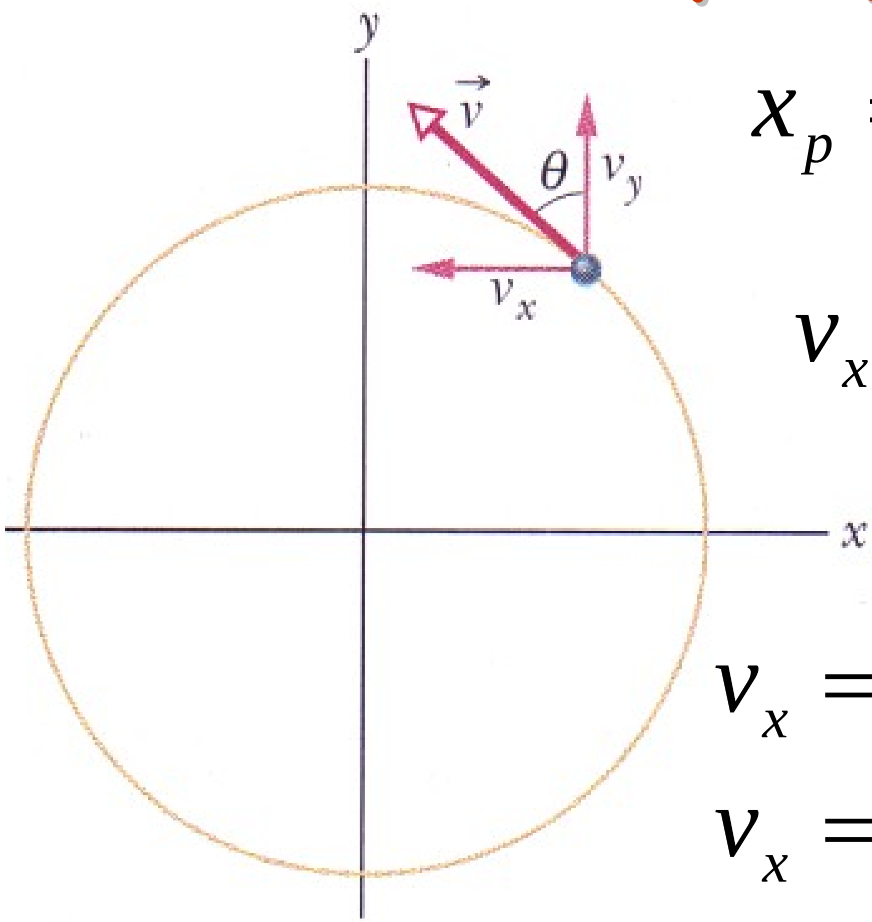
Uniform motion:

$$\theta = 2\pi\frac{t}{T} = \left(2\pi f\right)t = \omega t$$

$$\left[\text{Note: } 180^o = 2\pi \text{ radians}\right]$$

$$x_p = r\cos\omega t \qquad y_p = r\sin\omega t$$

Analyzing the motion



$$x_p = r\cos\omega t \qquad y_p = r\sin\omega t$$

$$v_x = \frac{dx_p}{dt} \qquad v_y = \frac{dy_p}{dt}$$

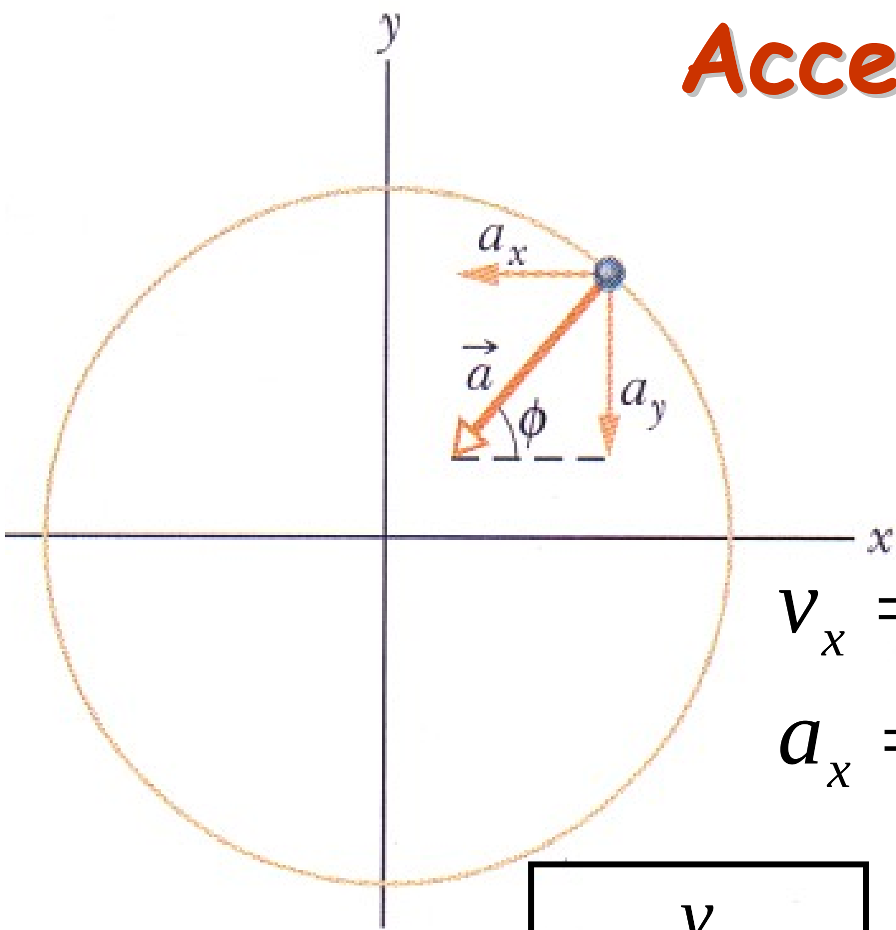
$$v_x = -r\omega\sin\omega t \quad v_y = r\omega\cos\omega t$$

$$v_x = -v\sin\omega t \qquad v_y = v\cos\omega t$$

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}}$$

$$v = r\omega \;\; [\text{remember}, \omega = \frac{v}{r}]$$

Acceleration



$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\frac{dv_x}{dt}\right)\hat{\mathrm{i}} + \left(\frac{dv_y}{dt}\right)\hat{\mathrm{j}}$$

$$= a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}$$

$$v_x = -v\sin\omega t \qquad v_y = v\cos\omega t$$

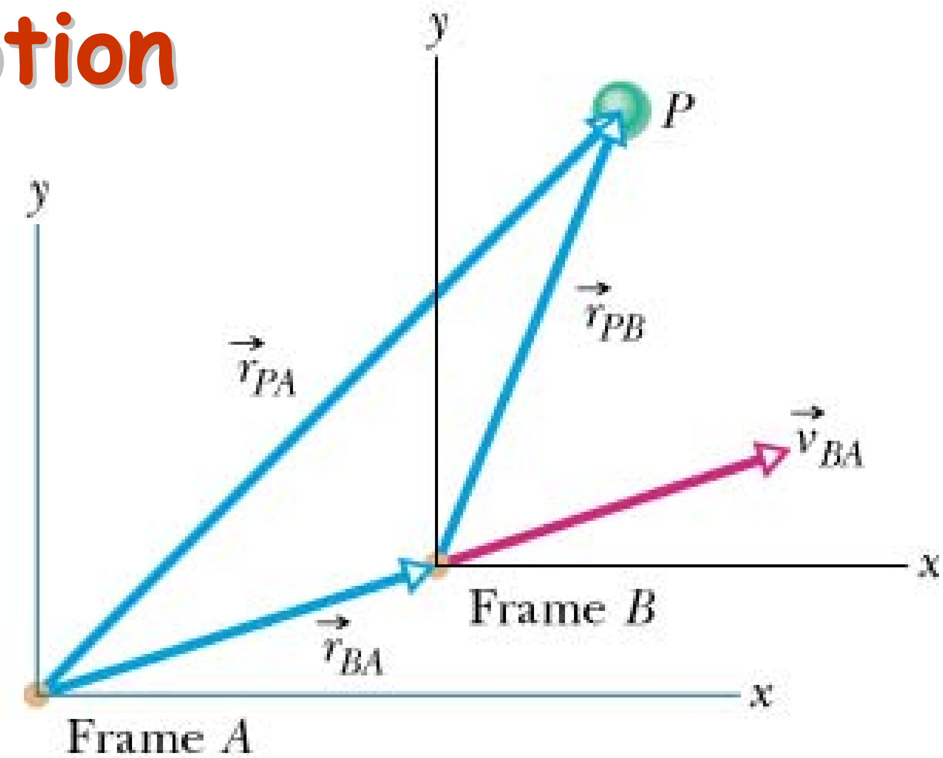
$$a_x = -v\omega\cos\omega t \quad a_y = -v\omega\sin\omega t$$

$$\boxed{\omega = \frac{v}{r} \quad \Rightarrow} \quad a_x = -\frac{v^2}{r}\cos\omega t \quad a_y = -\frac{v^2}{r}\sin\omega t$$

$$\left[\mathrm{recall}: x_p = r\cos\omega t \qquad y_p = r\sin\omega t\right]$$

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r} = v\omega$$

Relative motion



Position:

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

Velocity:

$$\vec{v}_{PA} = \frac{d\vec{r}_{PA}}{dt} = \frac{d\vec{r}_{PB}}{dt} + \frac{d\vec{r}_{BA}}{dt} = \vec{v}_{PB} + \vec{v}_{BA} = \vec{v}_{PB} + \vec{v}_{R}$$

Acceleration (if frame B is "inertial"):

$$\vec{a}_{PA} = \frac{d\vec{v}_{PA}}{dt} = \frac{d\vec{v}_{PB}}{dt} + \cancel{\frac{d\vec{v}_{R}}{dt}} = \vec{a}_{PB}$$