Chapter 1

Changing units

Chain-link conversion - an example:

1 minute = 60 seconds

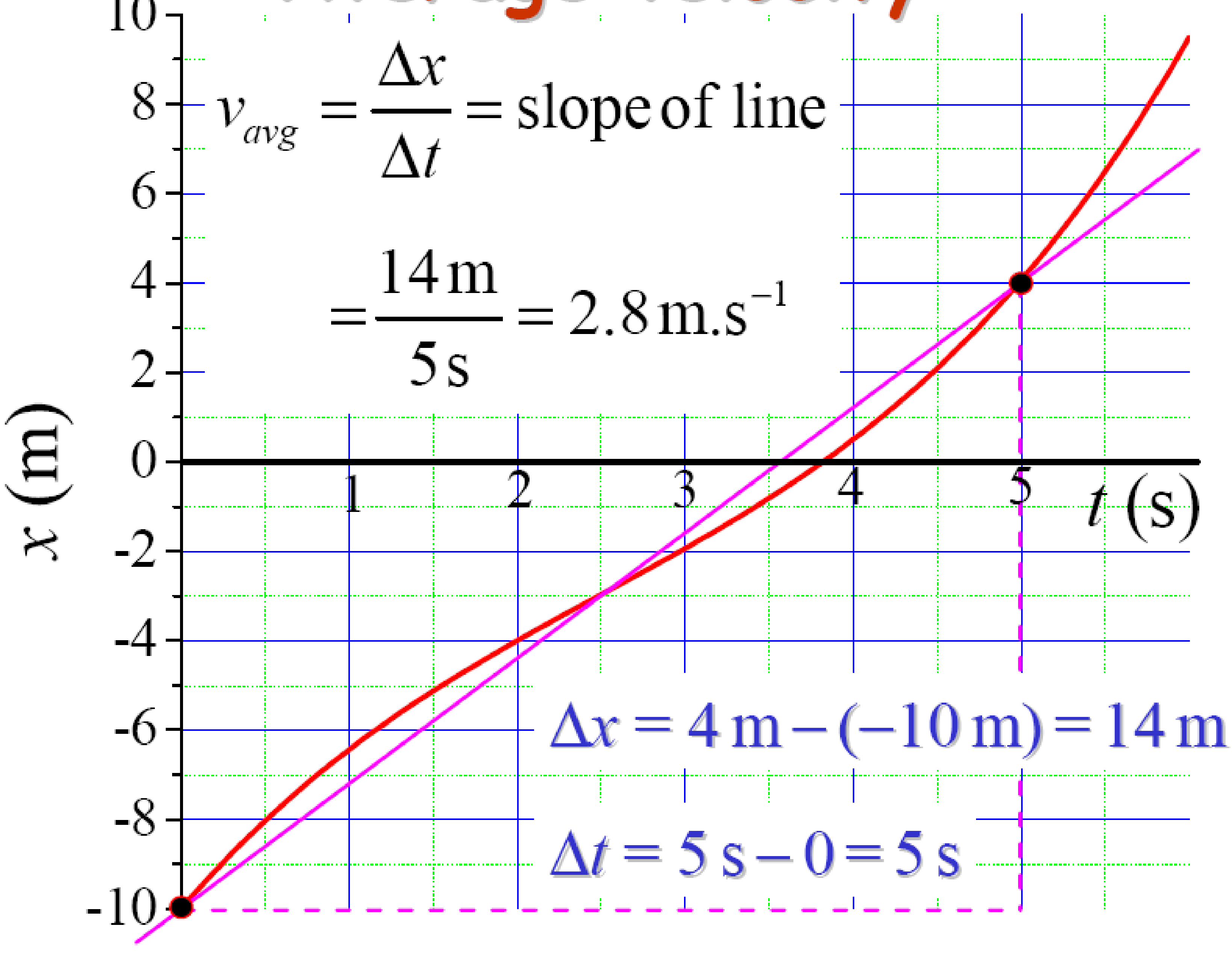
therefore $\dfrac{1\ \text{min}}{60\ \text{sec}} = 1$ or $\dfrac{60\ \text{sec}}{1\ \text{min}} = 1$

Note: this does not imply 60 = 1, or 1/60 = 1!

$$2\ \text{min} = (2\ \text{min}) \times (1) = (2\ \cancel{\text{min}}) \times \left(\frac{60\ \text{s}}{1\ \cancel{\text{min}}} \right) = 120\ \text{s}$$

Chapter 2

Average velocity



Average velocity and speed

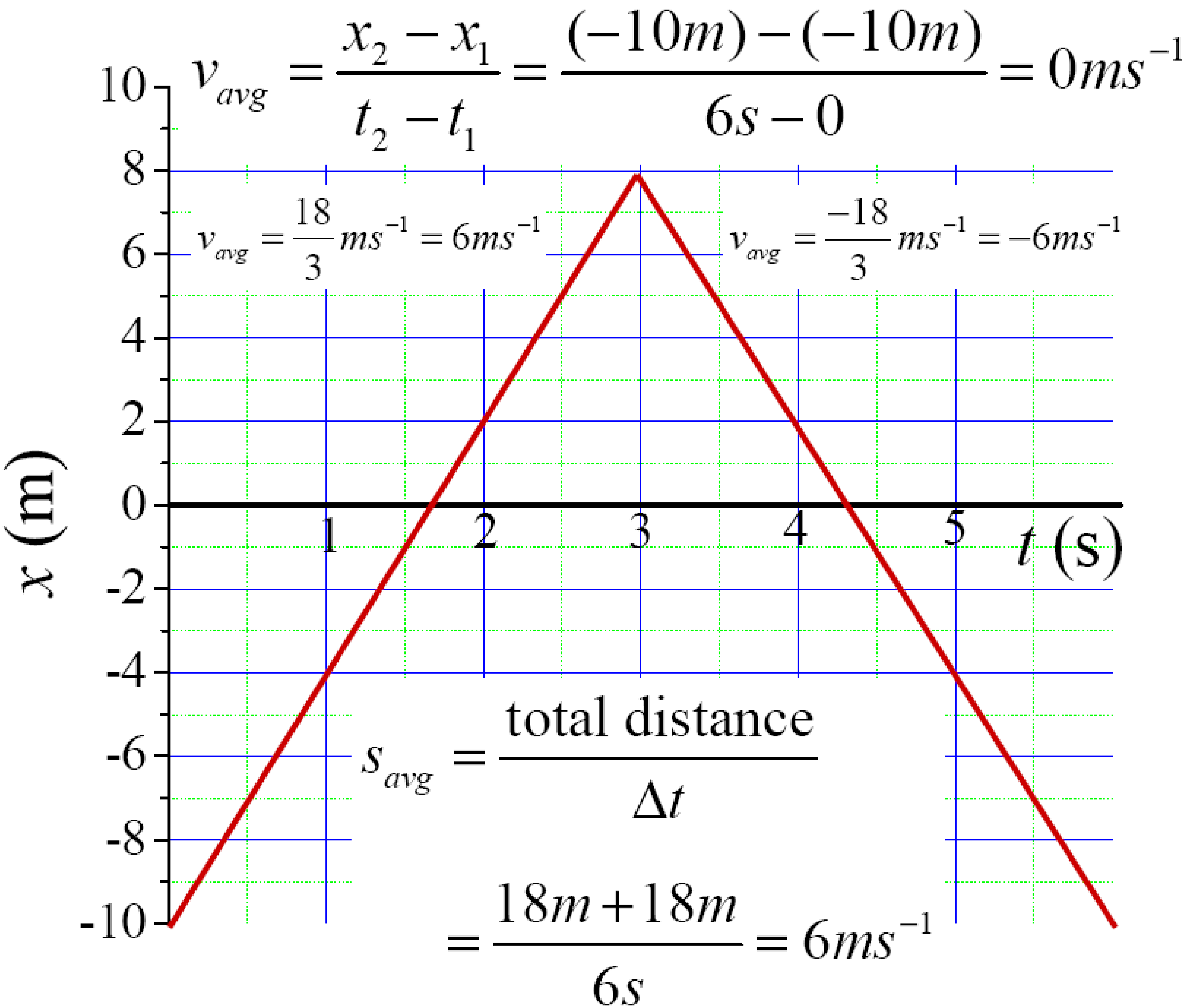
$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- Like displacement, v_{avg} is a vector

Average speed s_{avg}:

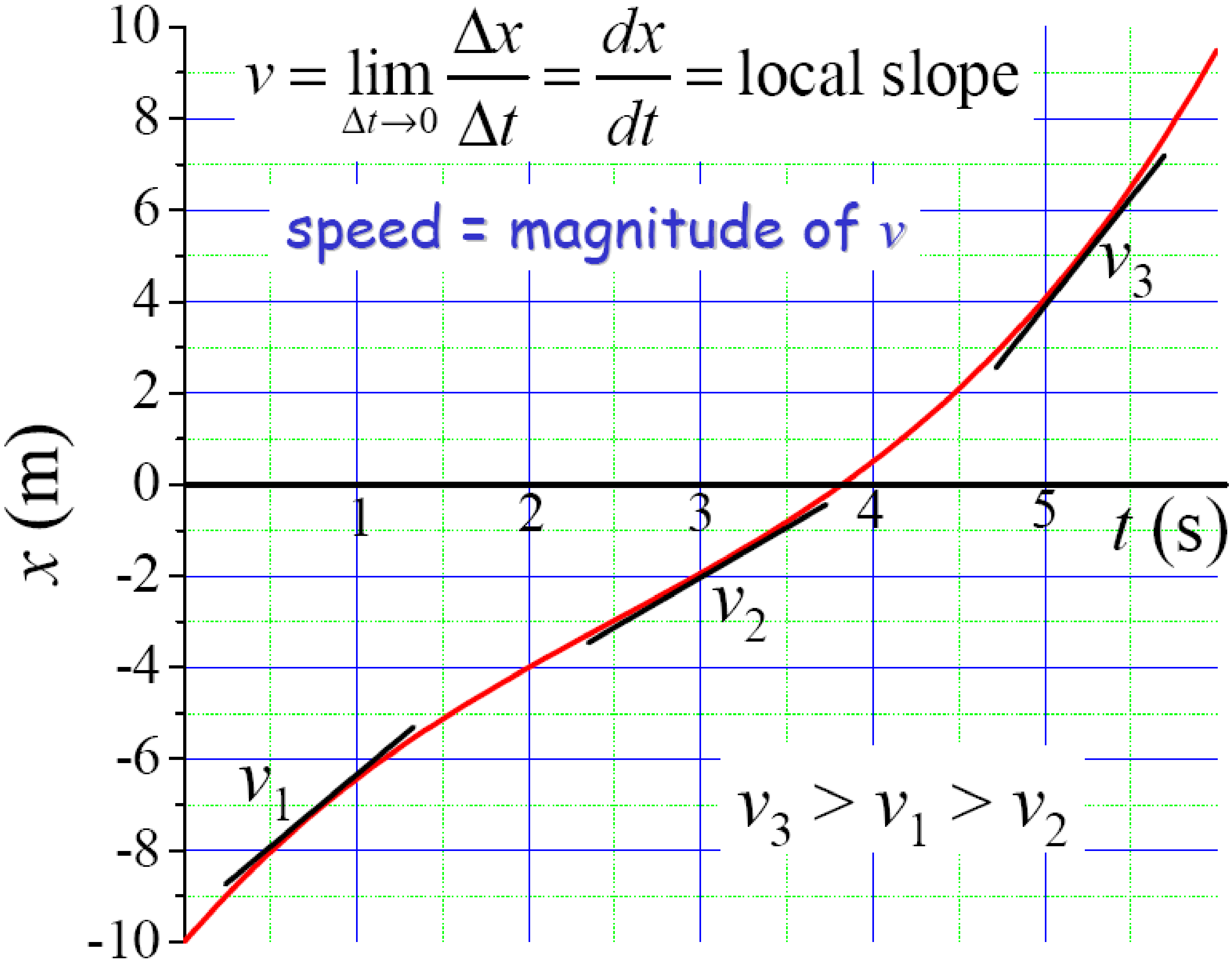
$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

- s_{avg} is not a vector - it lacks an algebraic sign
- How do v_{avg} and s_{avg} differ?



$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(-10m)-(-10m)}{6s - 0} = 0ms^{-1}$
$v_{avg} = \frac{18}{3} ms^{-1} = 6ms^{-1}$
$v_{avg} = \frac{-18}{3} ms^{-1} = -6ms^{-1}$
$s_{avg} = \frac{\text{total distance}}{\Delta t}$
$= \frac{18m + 18m}{6s} = 6ms^{-1}$
x (m)
t (s)
10
8
6
4
2
0
-2
-4
-6
-8
-10
1
2
3
4
5

Instantaneous velocity and speed



Acceleration

- An object is accelerating if its velocity is changing

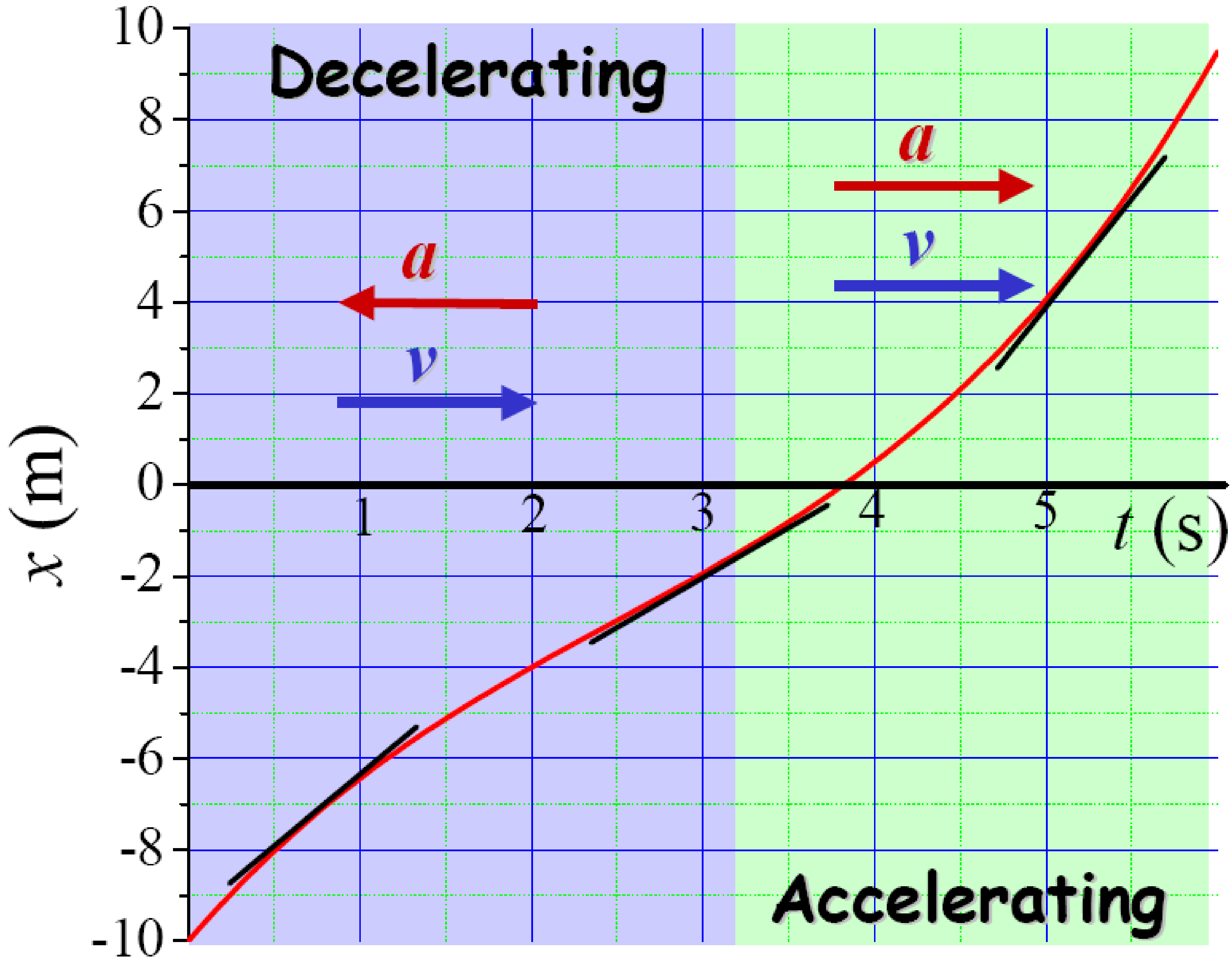
Average acceleration a_{avg}:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous acceleration a:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

- This is the second derivative of the x vs. t graph
- Like x and v, acceleration is a vector
- Note: direction of a need not be the same as v



Decelerating
Accelerating
a
v
a
v
x (m)
t (s)
10
8
6
4
2
0
-2
-4
-6
-8
-10
1
2
3
4
5

Constant acceleration: a special case

For non-calculus derivation, see section 2-6 in HRW

$$a = \frac{dv}{dt} \qquad \Rightarrow \qquad dv = a\,dt$$

$$\int dv = a \int dt \qquad \Rightarrow \qquad v = at + C$$

• To evaluate the constant of integration C, we let $v = v_o$ at time $t = 0$, i.e. v_o represents the initial velocity.

$$\Rightarrow \qquad \boxed{v = v_o + at \qquad (2-11)}$$

• Taking the derivative of Eq. 2-11 (v_o and a are constants), we recover:

$$a = \frac{dv}{dt}$$

Further integration

$$v = \frac{dx}{dt} \qquad \Rightarrow \qquad dx = v\,dt$$

$$\Rightarrow \qquad \int dx = \int v(t)\,dt$$

- Note - in general, v depends on t. However, we can substitute v from Eq. 2-11:

$$\int dx = \int \left(v_o + at\right) dt \qquad \Rightarrow \qquad x = v_o t + \tfrac{1}{2} a t^2 + C'$$

- To evaluate the constant of integration C', we let $x = x_o$ at time $t = 0$, i.e. x_o represents the initial position.

$$\Rightarrow \qquad \boxed{x - x_o = v_o t + \tfrac{1}{2} a t^2 \qquad (2-15)}$$

Constant acceleration formulae

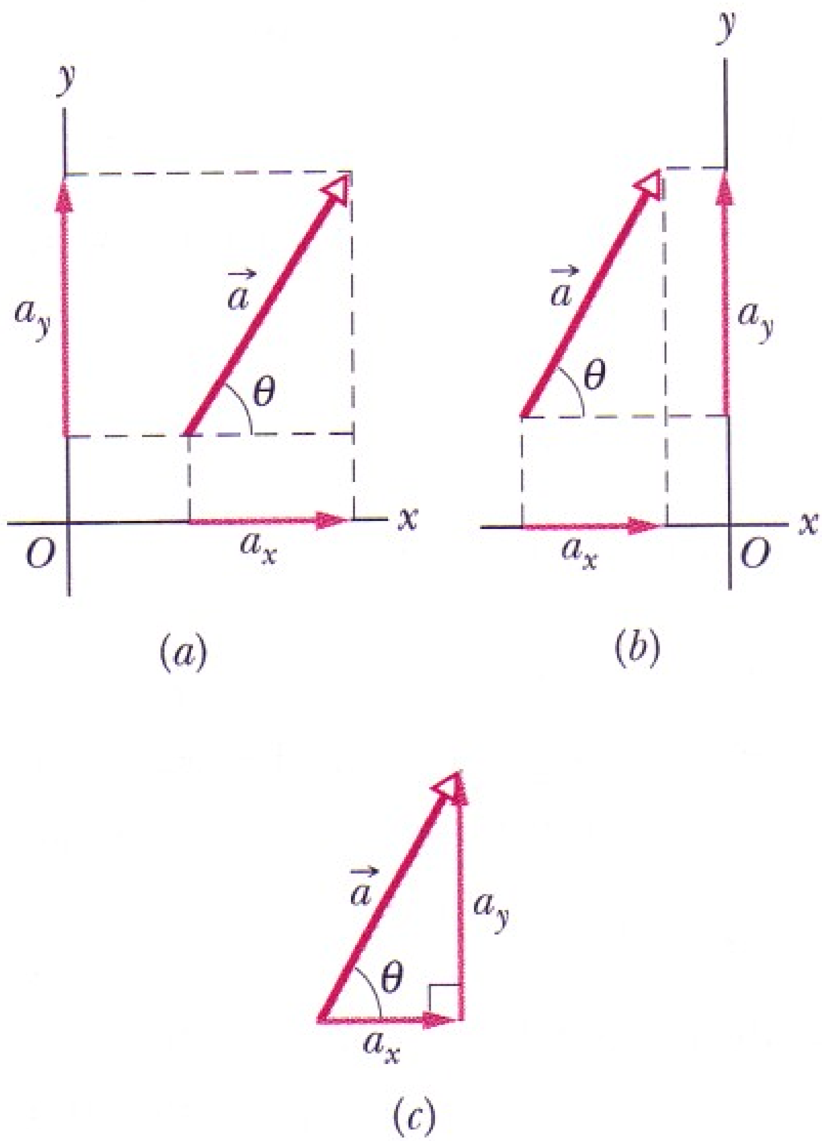
TABLE 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a] Make sure that the acceleration is indeed constant before using the equations in this table.

Chapter 3

Components of vectors



Resolving vector components

$$a_x = a\cos\theta$$

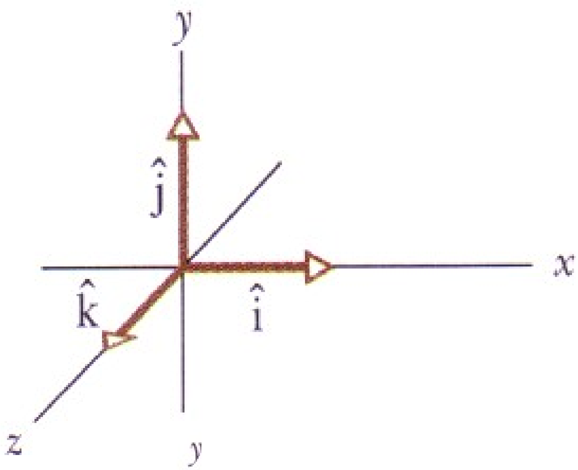
$$a_y = a\sin\theta$$

The inverse process

$$a = \sqrt{a_x^2 + a_y^2}$$

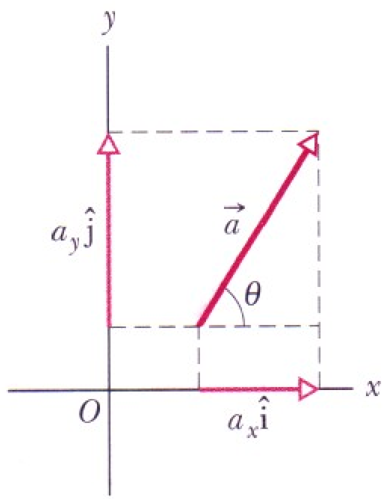
$$\tan\theta = \frac{a_y}{a_x}$$

Unit vectors



$\hat{\mathrm{i}}, \hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$ are unit vectors

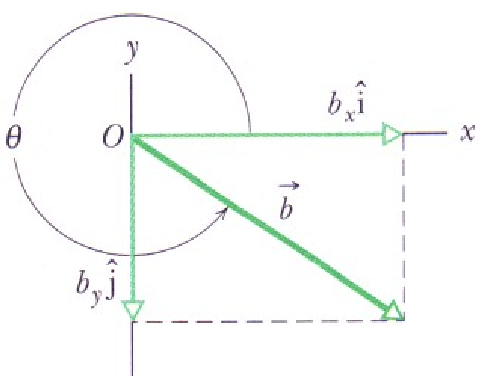
They have length equal to unity, and they point respectively along the *x*, *y* and *z* axes of a right handed orthogonal coordinate system



(*a*)

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}$$

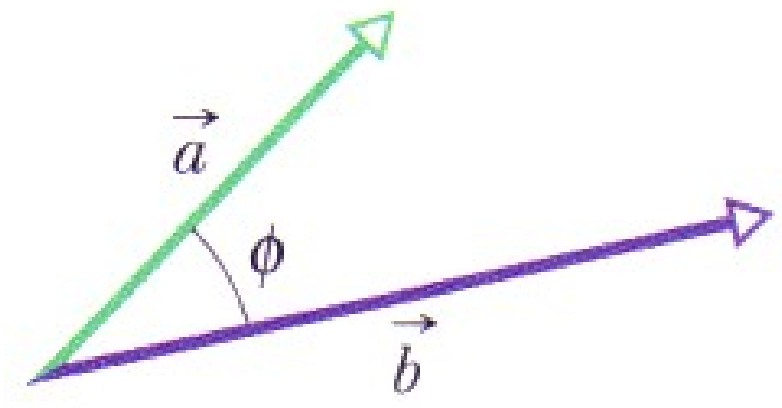
$$\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}$$



Note: θ is always measured in a right handed sense around the *z*-axis.

The scalar product, or dot product

$$\vec{a}\cdot\vec{b} = ab\cos\phi$$



$$(a)(b\cos\phi) = (a\cos\phi)(b)$$

$$\cos\phi = \cos(-\phi)$$

$$\Rightarrow \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

- They scalar product represents the product of the magnitude of one vector and the component of the second vector along the direction of the first

$$\text{If } \phi = 0^o, \text{ then } \vec{a}\cdot\vec{b} = ab$$

$$\text{If } \phi = 90^o, \text{ then } \vec{a}\cdot\vec{b} = 0$$

The scalar product in component form

$$\vec{a}\cdot\vec{b}=\left(a_x\hat{\mathrm{i}}+a_y\hat{\mathrm{j}}+a_z\hat{\mathrm{k}}\right)\cdot\left(b_x\hat{\mathrm{i}}+b_y\hat{\mathrm{j}}+b_z\hat{\mathrm{k}}\right)$$

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z$$

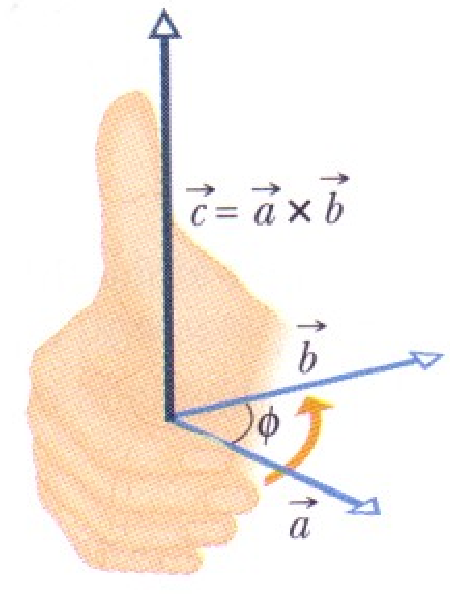
Because:

$$\hat{\mathrm{i}}\cdot\hat{\mathrm{i}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{k}}=1$$

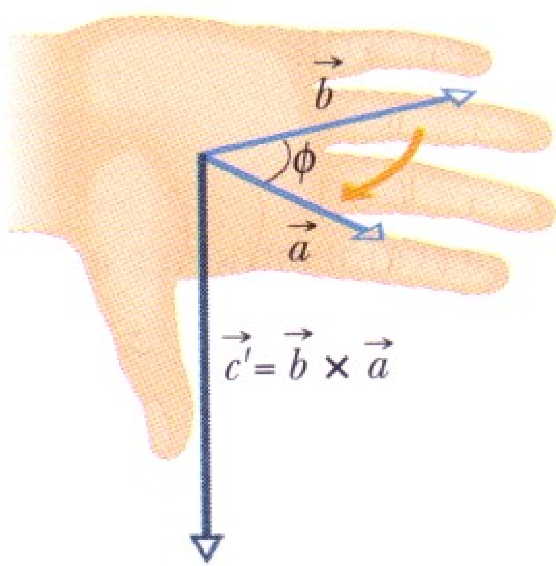
$$\hat{\mathrm{i}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{k}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{i}}=0$$

This is the property of orthogonality

The vector product, or cross product



(a)



$$\vec{a} \times \vec{b} = \vec{c}, \text{ where } c = ab\sin\phi$$

$$\vec{a} \times \vec{b} = -\left(\vec{b} \times \vec{a}\right)$$

Direction of $\vec{c} \perp$ to both $\vec{a}$ and $\vec{b}$

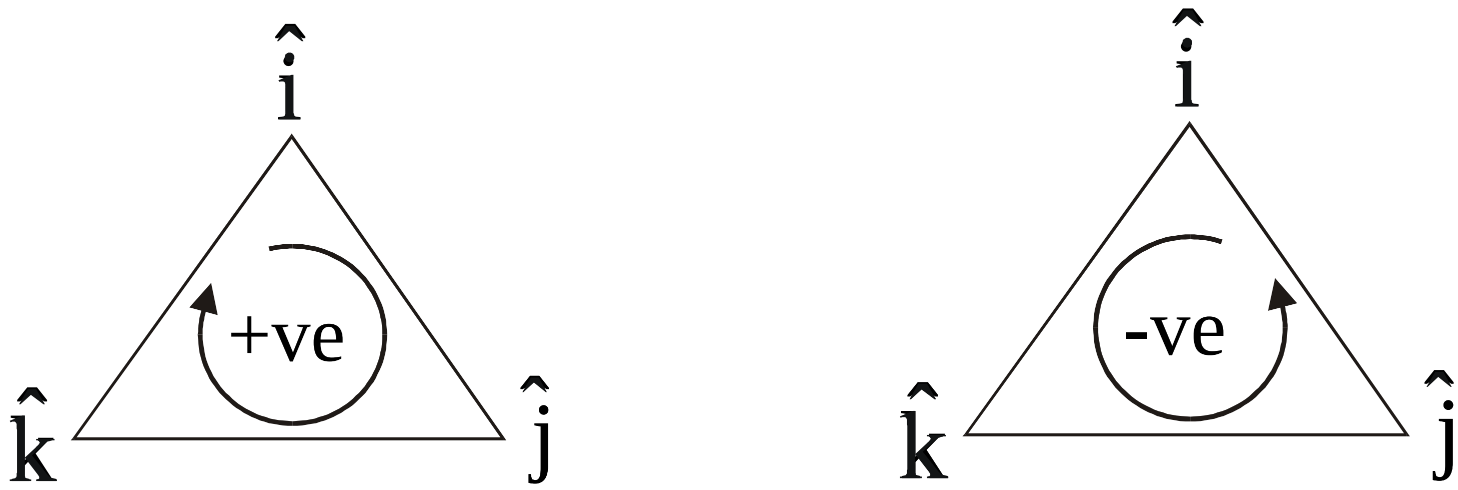
$$\hat{\mathrm{i}} \times \hat{\mathrm{i}} = \hat{\mathrm{j}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0$$

$$\hat{\mathrm{i}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}} \qquad \hat{\mathrm{j}} \times \hat{\mathrm{i}} = -\hat{\mathrm{k}}$$

$$\hat{\mathrm{j}} \times \hat{\mathrm{k}} = \hat{\mathrm{i}} \qquad \hat{\mathrm{k}} \times \hat{\mathrm{j}} = -\hat{\mathrm{i}}$$

$$\hat{\mathrm{k}} \times \hat{\mathrm{i}} = \hat{\mathrm{j}} \qquad \hat{\mathrm{i}} \times \hat{\mathrm{k}} = -\hat{\mathrm{j}}$$

$$\vec{a} \times \vec{b} = \left(a_x \hat{\mathrm{i}} + a_y \hat{\mathrm{j}} + a_z \hat{\mathrm{k}} \right) \times \left(b_x \hat{\mathrm{i}} + b_y \hat{\mathrm{j}} + b_z \hat{\mathrm{k}} \right)$$



$$a_x \hat{\mathrm{i}} \times b_y \hat{\mathrm{j}} = a_x b_y \left(\hat{\mathrm{i}} \times \hat{\mathrm{j}} \right) = a_x b_y \hat{\mathrm{k}}$$

$$\vec{a} \times \vec{b} = \left(a_y b_z - b_y a_z \right) \hat{\mathrm{i}} + \left(a_z b_x - b_z a_x \right) \hat{\mathrm{j}} + \left(a_x b_y - a_y b_x \right) \hat{\mathrm{k}}$$

Chapter 4

Projectile equations of motion

$$x - x_0 = (v_0 \cos\theta_0)t \qquad 4-21$$

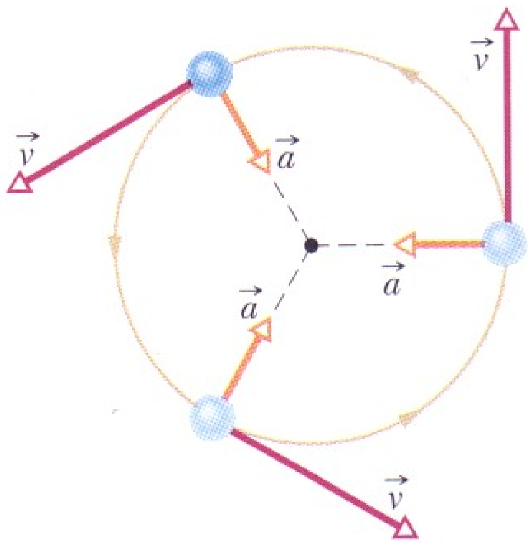
$$v_x = v_0 \cos\theta_0$$

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2 \qquad 4-22$$

$$v_y = v_0 \sin\theta_0 - gt \qquad 4-23$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0) \qquad 4-24$$

Uniform circular motion

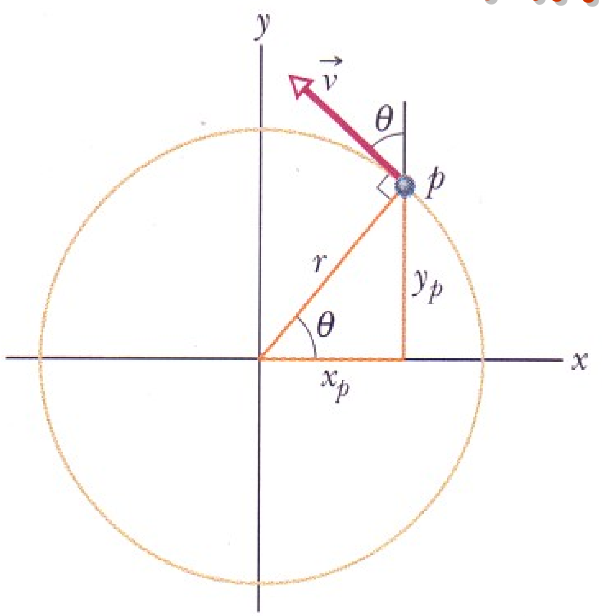


- Although v does not change, the direction of the motion does, *i.e.* the velocity (a vector) changes.
- Thus, there is an acceleration associated with the motion.
- We call this a centripetal acceleration.

Acceleration: $a = \frac{v^2}{r}$ Period: $T = \frac{2\pi r}{v}$

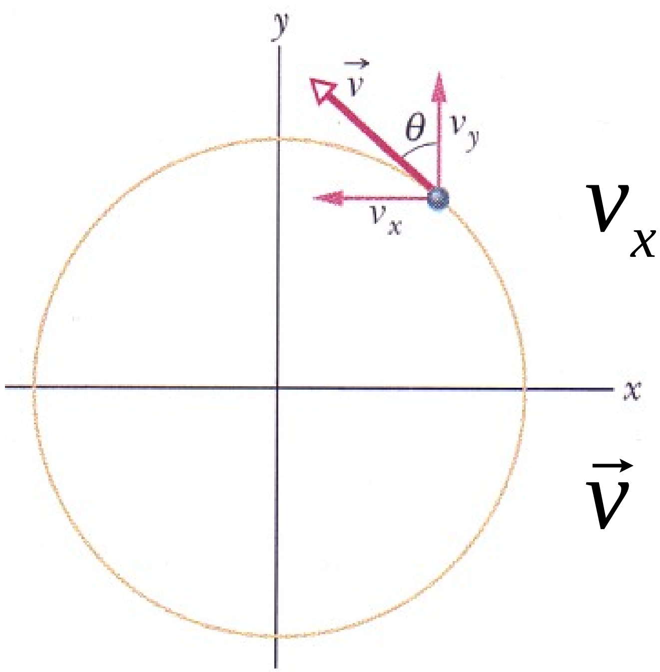
- Since v does not change, the acceleration must be perpendicular to the velocity.

Analyzing the motion



$$x_p = r\cos\theta \qquad y_p = r\sin\theta$$

$$\vec{r} = x_p\hat{\mathrm{i}} + y_p\hat{\mathrm{j}}$$

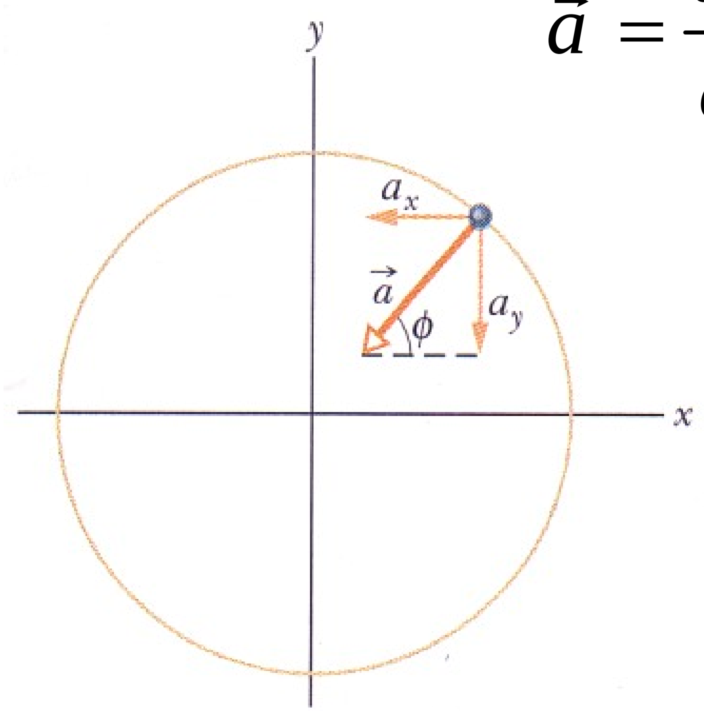


$$v_x = -v\sin\theta \qquad v_y = v\cos\theta$$

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = \left(-\frac{vy_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{vx_p}{r}\right)\hat{\mathrm{j}}$$

Acceleration

$$\left[\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = \left(-\frac{vy_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{vx_p}{r}\right)\hat{\mathrm{j}}; \quad v_x = -v\sin\theta \qquad v_y = v\cos\theta\right]$$



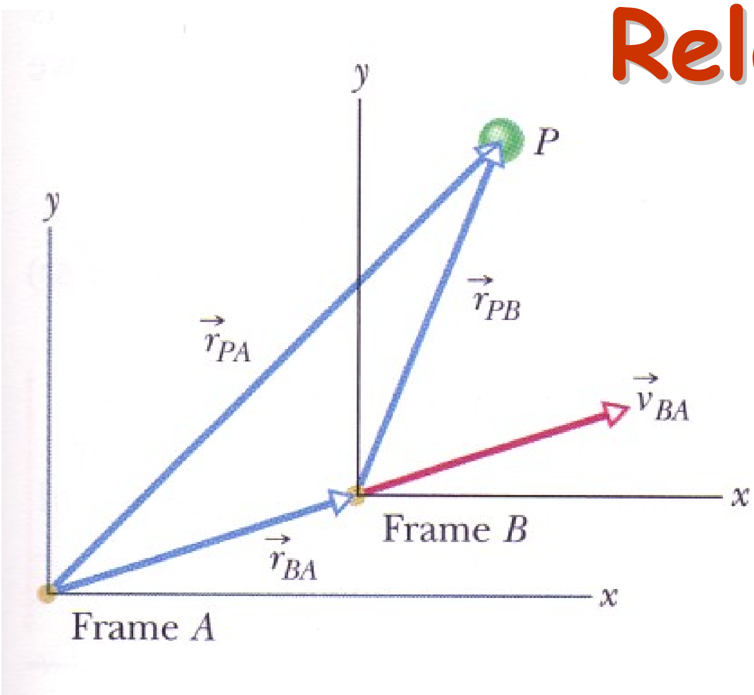
$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}$$

$$= \left(-\frac{v}{r}v_y\right)\hat{\mathrm{i}} + \left(\frac{v}{r}v_x\right)\hat{\mathrm{j}}$$

$$= \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}$$

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}$$

Relative motion



Position:

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

Velocity:

$$\vec{v}_{PA} = \frac{d\vec{r}_{PA}}{dt} = \frac{d\vec{r}_{PB}}{dt} + \frac{d\vec{r}_{BA}}{dt} = \vec{v}_{PB} + \vec{v}_{BA}$$

Acceleration:

$$\vec{a}_{PA} = \frac{d\vec{v}_{PA}}{dt} = \frac{d\vec{v}_{PB}}{dt} + \cancel{\frac{d\vec{v}_{BA}}{dt}} = \vec{a}_{PB}$$