Review - II (chapters 5 and 6)

Newton's 1st law: If no force acts on a body, then the body's velocity cannot change; that is, it cannot accelerate.
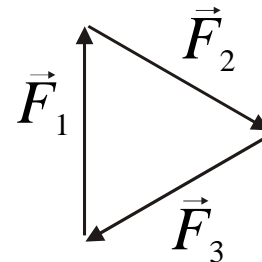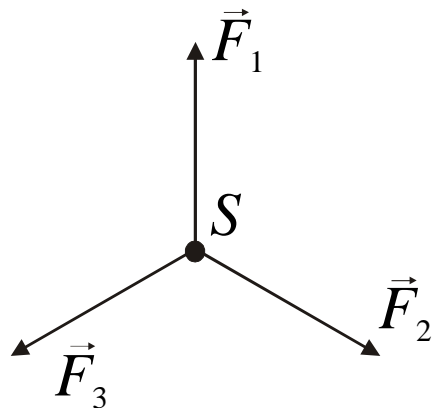
1 Newton is that force required to accelerate our standardized mass (1 Kg) at a rate of 1 m.s^{-2}.

Mass is simply the characteristic of a body that relates a force on the body to the resulting acceleration

Newton's 2nd law:

$$\vec{F}_{net} = m\vec{a}$$

Free-body diagrams

$\vec{F}_1$

S

$\vec{F}_2$

$\vec{F}_3$

$\vec{F}_1$

$\vec{F}_2$

$\vec{F}_3$
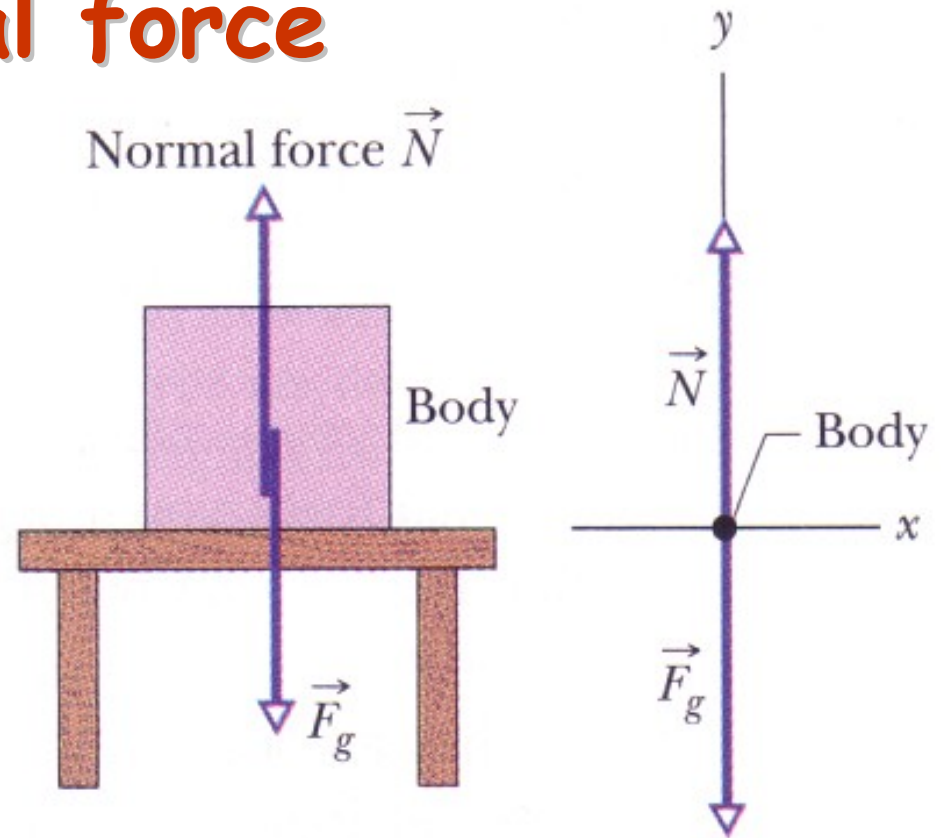
$$\sum \vec{F} = 0 = \vec{a}$$

Normal force

$$\vec{N} + \vec{F}_g = m\vec{a}$$

$$N - F_g = ma_y$$

$$N = ma_y + F_g = m(a_y + g)$$

$$N = F_g = mg \text{ if } a_y = 0$$

Normal force $\vec{N}$

Body

$\vec{N}$

Body

$\vec{F}_g$

$\vec{F}_g$

Weight (a force!):

• In the above example, the internal forces within the table supply the **normal force**, which is **normal to the surface**.

• If we hold the mass in a stationary state, we must supply the force. This is the sensation of weight, i.e.

$$W = F_g = mg \text{ Newtons}$$

Friction and tension

- We will deal with friction next week (chapter 6). All you need to know that a friction force acts parallel to a surface in the opposite direction to the motion.
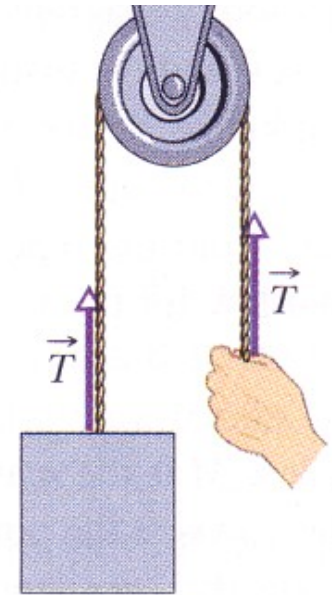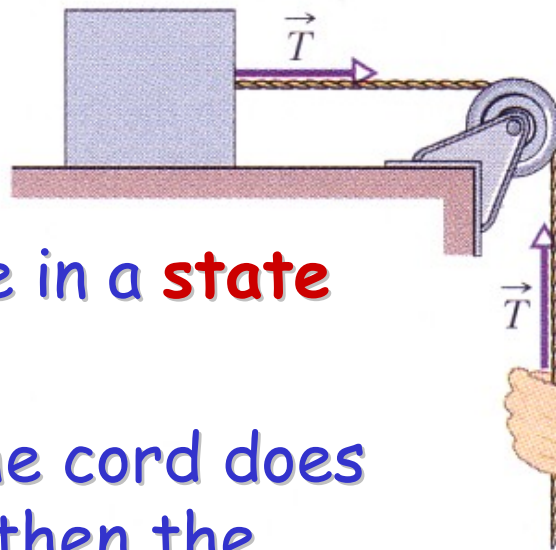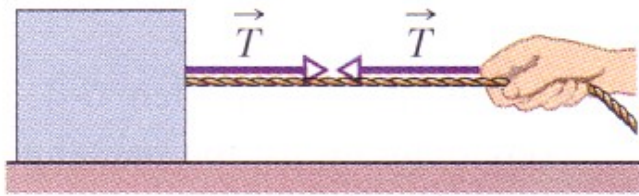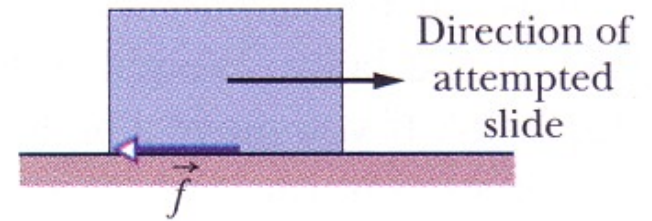
- A **taut cord** is said to be in a **state of tension**.

- If the body pulling on the cord does so with a force of 50 N, then the tension in the cord is 50 N.

- A taut cord pulls on objects at either end with equal and opposite force equal to the tension.

- Cords are massless, pulleys are massless and frictionless

Newton's 3rd law

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

For every **"action"** force, there is always an equal and opposite **"reaction"** force; we call these a **"third-law force pair."**

• When a table supports an object against the force of gravity, the internal forces within the table supply an upward **normal force**, which is **normal to the surface**.

• If we hold the mass in a stationary state, we must supply the normal force. This is the sensation of **weight**, i.e.

$$W = F_g = mg \text{ Newtons}$$

Review of static friction

1. In static situations, the static frictional force exactly cancels the component of the applied force parallel to the surface.

2. There is a maximum static frictional force which depends on the normal force between the surface and the object, i.e.

$$f_{s,\max} = \mu_s N$$

where μ_s is the **coefficient of static friction** and N is the magnitude of the normal force. μ_s is a parameter that depends on both surfaces. Once the force component parallel to the surface exceeds $f_{s,max}$, then the body begins to slide along the surface.

Review of kinetic friction

3. If a body begins to slide along the surface, the magnitude of the frictional force instantly decreases to a value f_k given by

$$f_k = \mu_k N$$

where μ_k is the **coefficient of kinetic friction** and N is the magnitude of the normal force. Therefore, during the sliding, a kinetic frictional force of magnitude f_k opposes the motion.

4. When several agents push in different directions on an object, the frictional force opposes the component of the net force on the object which is parallel to the surface.

Review of Drag force and terminal speed

$$D = \tfrac{1}{2}C\rho Av^2$$

$\vec{D}_{MA}$

Mass

$\vec{F}^g_{ME}$
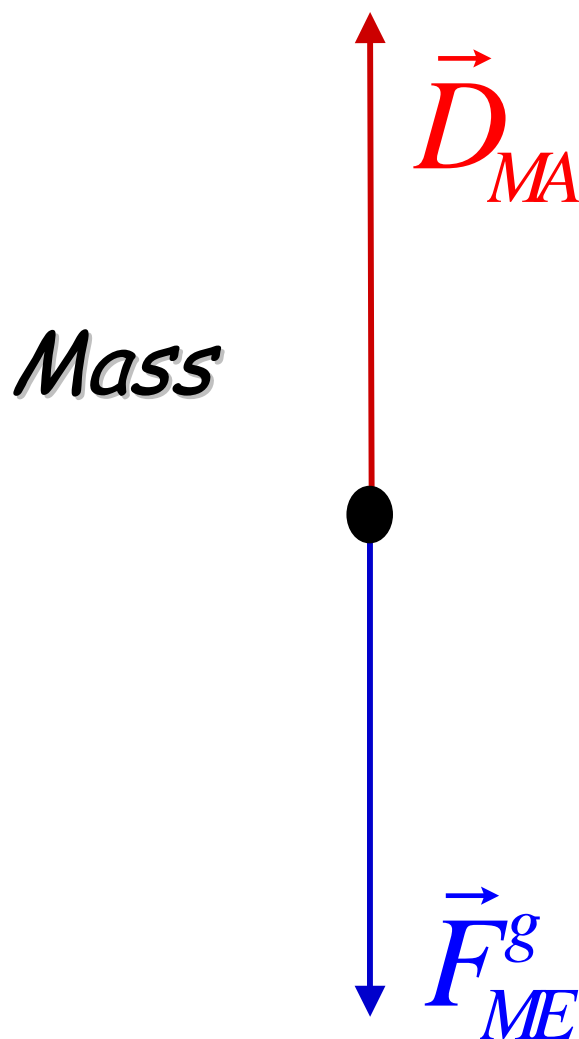
Newton's 2nd law:

$$D - F^g = \tfrac{1}{2}C\rho Av^2 - mg = ma$$

Terminal speed when $a = 0$.

$$\tfrac{1}{2}C\rho Av^2 = mg$$

or $\quad v = \sqrt{\dfrac{2mg}{C\rho A}}$

Review of uniform circular motion
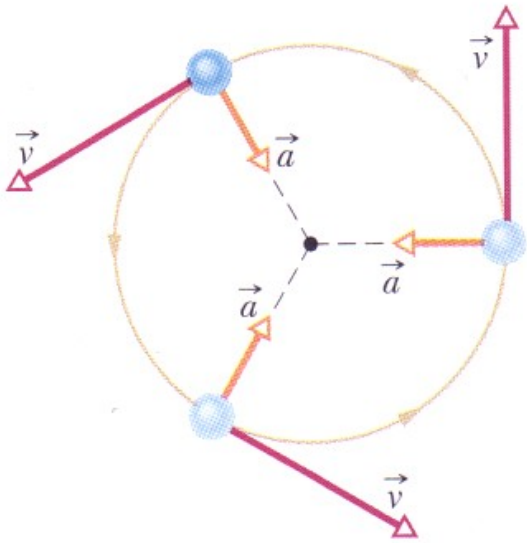
- Although v does not change, the direction of the motion does, i.e. the velocity (a vector) changes.

- Thus, there is an acceleration associated with the motion.

- We call this a centripetal acceleration.

$$a = \frac{v^2}{r} \qquad F = ma = \frac{mv^2}{r} \qquad \text{Period:} \quad T = \frac{2\pi r}{v}$$

- Since v does not change, the acceleration and force must be perpendicular to the velocity, i.e. directed towards the center of the motion.

Energy

Kinetic energy *K* is energy associated with the state of motion of an object. The faster an object moves, the greater its kinetic energy.

- Definition:

$$K = \tfrac{1}{2}mv^2$$

- SI unit is the joule (J):

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg.m}^2/\text{s}^2$$

Work

Work *W* is the energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

$$F_x = F\cos\phi; \qquad W = Fd\cos\phi; \qquad W = \vec{F}\cdot\vec{d}$$

Work done by a spring force

- Hooke's law: $$\vec{F} = -k\vec{d}$$

$\vec{d}$ is the displacement of the free end of the spring from its position when in a relaxed, or equilibrium state.

k is the **spring constant**, or **force constant**, and is a measure of the stiffness of the spring. It has dimensions of N.m^{-1}.

- Hooke's law (scalar version): $$F = -kx$$

$$\Rightarrow W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{work done by the spring}$$

Work is positive if the mass ends up closer to the relaxed position than it was initially. It is negative if the mass ends up further away.

If $x_i = 0$ and we call the final position x, then $W_s = -\frac{1}{2}kx^2$

Work done by an applied force

- This is very similar to the situation in which we do work against gravity.

- If we do work against a spring, then we do work on the spring, while the spring does work on us.

- If the end of the spring is stationary before and after a displacement, then

$$\Delta K = K_f - K_i = W_a + W_s = 0$$

$$\Rightarrow W_a = -W_s$$

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the displacing force is the negative of the work done on it by the spring.

Power

- Power is defined as the "rate at which work is done."

- If an amount of work W is done in a time interval Δt by a force, the average power due to the force during the time interval is defined as

$$P_{avg} = \frac{W}{\Delta t}$$

- Instantaneous power is defined as

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F} \cdot \right) = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

- The SI unit for power is the Watt (W).

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}$$

$$1 \text{ kilowatt-hour} = 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) = 3.60 \text{ MJ}$$

Energy conservation and potential energy

$$\Delta U = -W$$

• Note: the change in potential energy is simply the negative of the work done by the gravitational force.

• Therefore, we already know how to compute ΔU.

Mechanical energy

• We define mechanical energy as the sum of the kinetic and potential energy, i.e.

$$E_{mech} = K + U$$

• This is true only for ideal systems in which the only forces which act are what we call conservative forces.

• The only conservative forces which you will encounter in PHY2048 are gravitational and spring forces.

Calculation of potential energy

$$\Delta U = -W = -\int_{x_i}^{x_f} F(x)\,dx$$

Gravitational potential energy

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\,dy$$

$$= mg\left(y_f - y_i\right) = mg\,\Delta y$$

$$\Delta U = U - U_i = mg\left(y - y_i\right)$$

$$U = mgy$$

Elastic potential energy

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\,dx$$

$$= \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2$$

$$\Delta U = U - 0 = \tfrac{1}{2}kx^2 - 0$$

$$U = \tfrac{1}{2}kx^2$$

Conservation of mechanical energy

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can separately change, but their sum, the mechanical energy of the system, cannot change.

$$\Delta E_{mech} = \Delta U + \Delta K = 0$$

When the mechanical energy of a system is conserved, we can related the sum of kinetic and potential energy at one instant to that at another instant without consideration of the intermediate motion and without finding the work done by the forces involved.

$$U_2 + K_2 = U_1 + K_1$$

Calculation of force from potential energy

$$\Delta U = -W = -\int_{x_i}^{x_f} F(x)dx$$

•Working backwards, since ΔU is related to F through an integration, it should come as no surprise that F is related to U through differentiation.

$$\Delta U(x) = -W = -F(x)\Delta x$$

Yielding:

$$F(x) = -\frac{\Delta U(x)}{\Delta x}, \qquad \text{or} \qquad F(x) = -\frac{dU(x)}{dx}$$
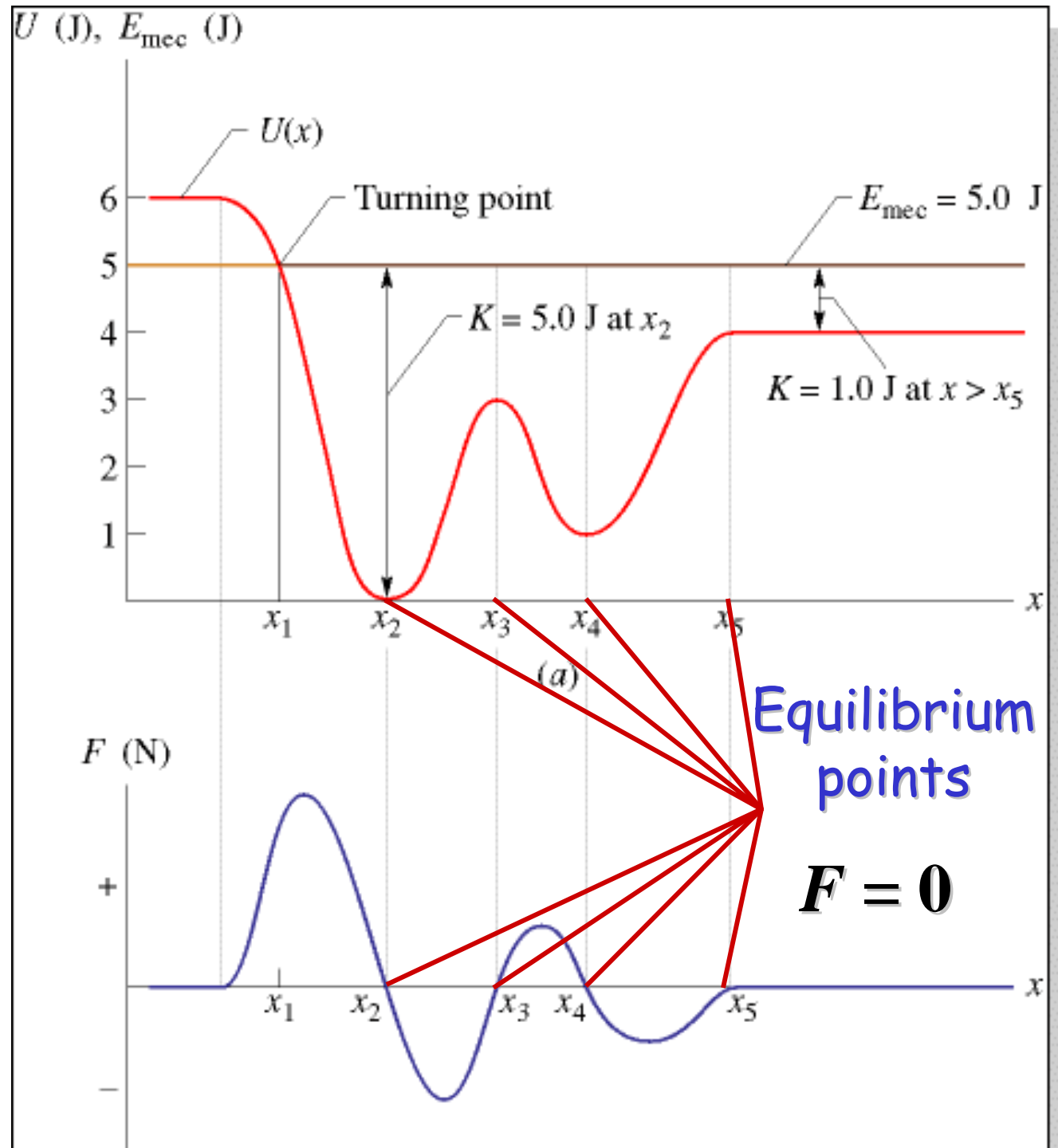
Potential energy curve

$$E_{mech} = K + U$$

• K cannot be negative, since it is proportional to (velocity)^2

• Therefore,

$$U \leq E_{mech}$$

always!!

U (J), E_{mec} (J)

$U(x)$

6

Turning point

$E_{mec} = 5.0$ J

5

4

$K = 5.0$ J at x_2

$K = 1.0$ J at $x > x_5$

3

2

1

x_1 x_2 x_3 x_4 x_5 x

(a)

F (N)

+

x_1 x_2 x_3 x_4 x_5 x

−

Equilibrium points

$F = 0$

Conservation of energy

The total energy of a system can change only by amounts of energy W that are transferred to or from the system.

$$W = \Delta E = \Delta E_{mech} + \Delta E_{th}$$

where ΔE_{th} acknowledges the fact that mechanical energy may be converted to thermal energy due to frictional forces or air resistance

The total energy of an isolated system cannot change.

$$\Delta E_{mech} + \Delta E_{th} = 0$$

In an isolated system, we can relate the total energy at one instant to the total energy at another instant without considering the energies at intermediate times.

Systems of particles

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots\cdots + m_n x_n}{M}$$

$$= \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$

•Here, i is a running number, or index, that takes on all integer values from 1 to n.

•In three-dimensions:

$$x_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i; \quad y_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i y_i; \quad z_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i z_i$$

$$\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + z_{com}\hat{k}$$

Linear momentum

- Definition of linear momentum, p:

$$\vec{p} = m\vec{v}$$

- If one takes the derivative,

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net}$$

The time rate of change of momentum of a particle is equal to the net force acting on the particle and is in the direction of the force.

Linear momentum of a system of particles

- A system of *n* particles has a total linear momentum given by:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots\cdots + \vec{p}_n$$

$$= m\vec{v}_1 + m\vec{v}_2 + m\vec{v}_3 + \cdots\cdots + m\vec{v}_n$$

$$= M\,\vec{v}_{com}$$

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

$$\frac{d\vec{P}}{dt} = M\,\frac{d\vec{v}_{com}}{dt} = M\vec{a} = \vec{F}_{net}$$

Conservation of linear momentum

- For a system of *n* particles, if no net force acts on the system:

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system)}$$

If no net external force acts on a system of particles, the total linear momentum of the system cannot change

$$\begin{pmatrix} \text{total linear momentum} \\[4pt] \text{at some initial time } t_i \end{pmatrix} = \begin{pmatrix} \text{total linear momentum} \\[4pt] \text{at some later time } t_f \end{pmatrix}$$

- These are vector equations, *i.e.*

$$P_x = \text{constant}; \quad P_y = \text{constant}; \quad P_z = \text{constant}$$

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.
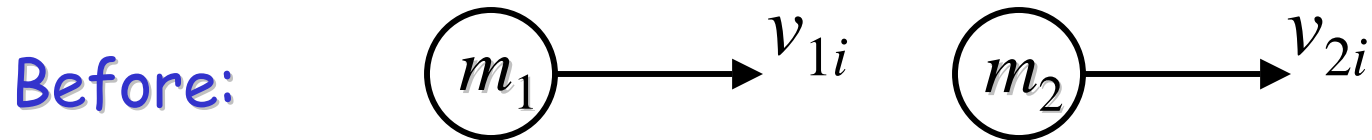
Completely inelastic collision - general case

Before:

m_1 → v_{1i} m_2 → v_{2i}

After:
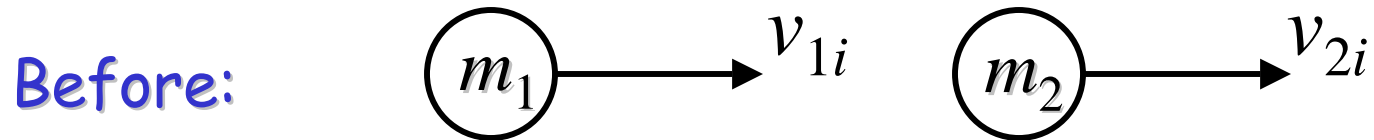
m_1 m_2 → v_f

$$\Rightarrow \quad v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Special case:

$$v_{2i} = 0$$

$$\Rightarrow \quad v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

Perfect elastic collision - general case

Before: m_1 →v_{1i} m_2 →v_{2i}

After: m_1 →v_{1f} m_2 →v_{2f}

General result:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

If $v_{2i} = 0$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$