10/11/04
PHY 2048

Remember ...

(1) Homework

* Chapter 8 ⇒ due Wed, Oct 13 @ 11pm
* Chapter 9 ⇒ due Sun, Oct 17 @ 11 pm
* 1/4 worked in class this week!

(2) Exam II, Tuesday, Oct. 19, 8:20 - 10:20 pm

* Chapters 5-9
* Get help NOW if you need it

(3) Review Session

* next Monday (Oct. 18)
* 6:15 - 8:10 pm
* Here

Center of Mass

System: $\left\{\begin{array}{l}\text{N particles}\\ \text{masses} \Rightarrow m_i\\ \text{positions} \Rightarrow \vec{x}_i\end{array}\right\}$ or $\left\{\begin{array}{l}\text{continuous}\\ \text{density}\\ \text{function } \rho(\vec{x})\end{array}\right\}$

Total Mass: $M \equiv \sum_{i=1}^{N} m_i$ or $\int dx \int dy \int dz\, \rho(x,y,z)$

COM: $\vec{X} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{x}_i$ or $\frac{1}{M} \int dx \int dy \int dz\, \vec{x}\, \rho(x,y,z)$

$$\boxed{\text{Little Steps for Little Feet}}$$

① 2 Particles in D=1

- $M = m_1 + m_2$
- $X = \frac{1}{M}(m_1 x_1 + m_2 x_2)$

② N particles in D=1

- $M = m_1 + m_2 + \dots + m_N$
- $X = \frac{1}{M}(m_1 x_1 + m_2 x_2 + \dots + m_N x_N)$

③ N particles in D=3

$$\left.\begin{array}{l} X = \frac{1}{M}(m_1 x_1 + \dots + m_N x_N) \\ Y = \frac{1}{M}(m_1 y_1 + \dots + m_N y_N) \\ Z = \frac{1}{M}(m_1 z_1 + \dots + m_N z_N) \end{array}\right\} \Rightarrow \vec{X} = \frac{m_1 \vec{x}_1 + \dots + m_N \vec{x}_N}{M}$$

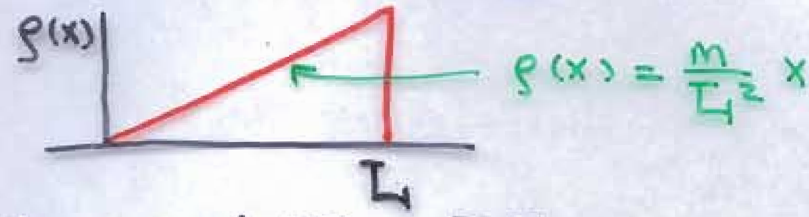
④ Density $\rho(x)$ from $x = L$ to $x = R$

- $M = \int_L^R dx\, \rho(x)$
- $X = \frac{1}{M} \int_L^R dx\, x\, \rho(x)$

More on COM

① Sometimes you must integrate

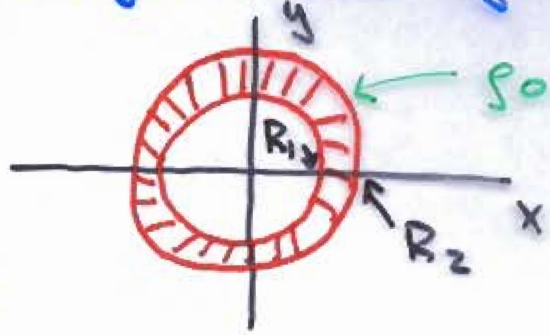
Eg $D=1$



$\Rightarrow M = \frac{1}{2} m$ and $X = \frac{2}{3} L_1$

② Sometimes you can just use symmetry

Eg $D=2$



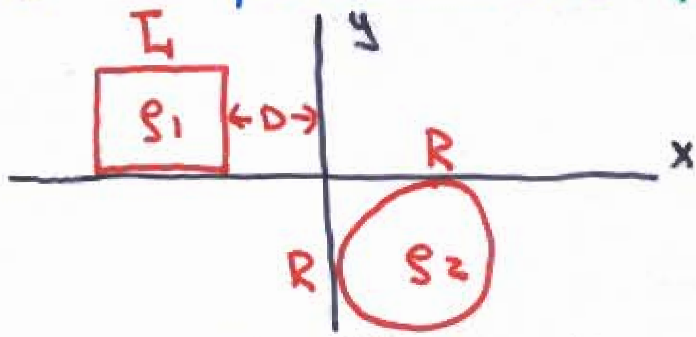
$\Rightarrow M = \pi (R_2^2 - R_1^2) \rho_0$

$X = 0$ & $Y = 0$

③ NB could be no mass at COM!

④ Treat "pieces" as point masses (cf #4 in Ch9)

Eg $D=2$



$M_1 = L_1^2 \rho_1$, $X_1 = -D - \frac{1}{2} L_1$, $Y_1 = +\frac{1}{2} L_1$

$M_2 = \pi R^2 \rho_2$, $X_2 = R$, $Y_2 = -R$

$\therefore M = M_1 + M_2$

$X = \frac{1}{M}(M_1 X_1 + M_2 X_2)$

$Y = \frac{1}{M}(M_1 Y_1 + M_2 Y_2)$

Newton's Law for Systems

① Just add $\vec{F} = m\vec{a} = \frac{d^2}{dt^2}(m\vec{x})$ $\forall$ piece!

* $\vec{F}_1 = \frac{d^2}{dt^2}(m_1 \vec{x}_1)$
* $\vec{F}_2 = \frac{d^2}{dt^2}(m_2 \vec{x}_2)$

$\vdots$

* $\vec{F}_N = \frac{d^2}{dt^2}(m_N \vec{x}_N)$

$$\vec{F}_1 + \dots \vec{F}_N = \frac{d^2}{dt^2}(m_1 \vec{x}_1 + \cdots + m_N \vec{x}_N)$$

② NB 3rd Law Forces cancel! ($\vec{F}_{i\,on\,j} = -\vec{F}_{j\,on\,i}$)

* cf Problem #68 in Chapter 6
* $\vec{F}_1 + \cdots + \vec{F}_N = \vec{F}_{ext}$

③ $m_1\vec{x}_1 + \dots + m_N\vec{x}_N = M * \frac{1}{M}(m_1\vec{x}_1 + \cdots + m_N\vec{x}_N)$

$$= M\vec{X}$$

$$\therefore \boxed{\vec{F}_{ext} = M\ddot{\vec{X}}}$$

What good is this ⇒ Problem #16 in Ch. 9!

Linear Momentum

$$\boxed{\vec{p} \equiv m\vec{v}}$$

cf #20 in Ch. 9

Soda Cans & Fire Crackers

$x_1(t) = -s_1 t$

$x_2(t) = +s_2 t$

* $M = m_1 + m_2$

* $X(t) = \dfrac{m_1 x_1(t) + m_2 x_2(t)}{M} = \dfrac{-m_1 s_1 t + m_2 s_2 t}{M} = 0$

$\therefore\ -m_1 s_1 + m_2 s_2 = p_1 + p_2 = 0$

Also note: $\vec{F} = m\vec{a} \Leftrightarrow \vec{F} = \frac{d}{dt}(\vec{p})$

Impulse

Suppose $\vec{F}(t) = 0$ except for $t_i \le t \le t_f$

Impulse: $\vec{J} \equiv \int_{t_i}^{t_f} dt\, \vec{F}(t)$

$\vec{F}(t) = \frac{d}{dt}(\vec{p}(t))$ Implies

1. $\vec{p}(t) = \vec{p}_i$ for $t < t_i$
2. $\vec{p}(t) = \vec{p}_f$ for $t_f < t$
3. $\vec{J} = \vec{p}_f - \vec{p}_i$

cF Problem #32 in Ch. 9