Chapter 15: Oscillations

Lecture 33
11/18/2009
Oscillations

Goals for this Lecture:

- Displacement, velocity and acceleration of a simple harmonic oscillator
- Energy of a simple harmonic oscillator
- Examples of simple harmonic oscillators: spring-mass system, simple pendulum, physical pendulum, torsion pendulum
- Damped harmonic oscillator
- Forced oscillations/Resonance
Oscillations

Oscillations are a periodic, repetitive motion (of any physical variable)

Examples:

- **Mechanical oscillations**: Heart beats, sound waves, earthquakes, swing set, rocking chair ...
- **Other oscillations**: Electro-magnetic, temperature stock market prices, # of students in this room, ...

![Graph showing oscillations of different variables over time: # student in room vs time, Heart beat EKG, Dow Jones Index.](image)
Simple Harmonic Motion (SHM) is a class of oscillations that is
- Periodic: repeats exactly with a period $T$ (or a frequency $f = 1/T$)
- Described by a sinusoidal function:
  $$x(t) = x_m \cos(\omega t + \Phi)$$
Properties of SHM

- **Properties:**
  - **Period:** $T$ [s]  \Rightarrow x(t) = x(t+T)
  - **Frequency:** $f$ [Hz] = $[s^{-1}]$  \Rightarrow f = 1/T
  - **Angular frequency:** $\omega$ [s$^{-1}$]  \Rightarrow \omega = 2\pi f = 2\pi/T
  - **Amplitude:** $x_m$ [m]
  - **Phase:** $(\omega t + \Phi)$ [rad]
  - **Phase constant:** $\Phi$ [rad]

Two different frequencies
Example

The displacement of a mass particle at \( t=0 \) is 5 m. If the maximum displacement is 10 m find the phase constant (\( \Phi \)) of this motion.

\[
x(t) = x_m \cos(\omega t + \Phi)
\]

\[
x(0) = 5m = 10m \cos(\Phi)
\]

\[
\cos(\Phi) = 0.5
\]

\[
\Phi = \cos^{-1} 0.5
\]
Velocity of a SHM

For a simple harmonic oscillator we have

- Position: \( x(t) = x_m \cos(\omega t + \Phi) \)
- Velocity: \( v(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \Phi) \)
  
  \[ = -v_m \sin(\omega t + \Phi), \quad v_m = \omega x_m \]
  
  \[ = v_m \cos(\omega t + \Phi + \pi/2) \]
Acceleration of a SHM

For a simple harmonic oscillator we have

- **Position:** \( x(t) = x_m \cos(\omega t + \Phi) \)
- **Velocity:** \( v(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \Phi) \)
- **Acceleration:** \( a(t) = \frac{d^2x}{dt^2} = -\omega^2 x_m \cos(\omega t + \Phi) \)
  \[= -a_m \cos(\omega t + \Phi), \quad a_m = \omega^2 x_m \]
  \[= a_m \cos(\omega t + \Phi + \pi) \]

[Graph showing time, position, and acceleration functions]
Comparing $x(t)$, $v(t)$, $a(t)$
The Force Law for SHM

Consider a mass undergoing simple harmonic motion:
- Position: $x(t) = x_m \cos(\omega t + \Phi)$
- Acceleration: $a(t) = -\omega^2 x_m \cos(\omega t + \Phi) = -\omega^2 x(t)$

The force acting on the mass is
- $F = ma(t) = -m\omega^2 x(t)$

The force is proportional to the displacement $x$
- Its direction is opposite to the displacement $x$

..... this sounds suspiciously familiar ...
The Simple Harmonic Oscillator

- The force $F$ is proportional to the displacement $x$
- The frequency $\omega$ only depends on $k$ and $m$
- It DOES NOT depend on:
  - the size of the initial displacement
  - the initial speed
  - the phase constant
- A stiff spring (large $k$) and a small mass ($m$) result in a high oscillation frequency $\omega$

\[ F_{\text{spring}} = -kx \]
\[ F_{\text{SHM}} = -m\omega^2x \]
\[ \Rightarrow k = m\omega^2 \]
\[ \omega = \sqrt{\frac{k}{m}} \]
\[ T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \]
Bouncy mattress

A 50 kg kid jumps up and down on his bed’s mattress, undergoing simple harmonic motion. The period of the motion is 2 s and the amplitude of oscillations are 10 cm.

a) What is the magnitude of the maximum force acting on the kid?

b) What is the spring constant $k$ of the mattress springs?
Bouncy mattress

a) What is the magnitude of the maximum force acting on the kid?

\[ x(t) = x_m \cos(\omega t + \Phi) \]
\[ a(t) = -\omega^2 x_m \cos(\omega t + \Phi) \]
\[ F(t) = ma(t) = -m\omega^2 x_m \cos(\omega t + \Phi) \]
\[ F_{\text{max}} = m\omega^2 x_m \]
\[ \omega = 2\pi/T = 2\pi/2s = 3.14 \text{ s}^{-1} \]
\[ F_{\text{max}} = (50 \text{ kg})(\pi \text{ s}^{-1})^2 (0.1 \text{ m}) = 49.3 \text{ kg m/s}^2 = 49.3 \text{ N} \]

b) What is the spring constant \( k \) of the mattress springs?

\[ F_{\text{spring}} = -kx, \ F_{\text{SHM}} = -m\omega^2 x \]
\[ \Rightarrow k = m\omega^2 = (50 \text{ kg})(\pi \text{ s}^{-1})^2 = 493 \text{ kg/s}^2 = 493 \text{ N/m} \]