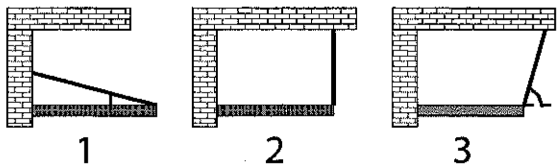
I for a uniform disk or cylinder around its axis of symmetry = 0.5 MR^2

Take g = 10 m/s^2 as the acceleration due to gravity ρ_{water} **= 1000 kg/m^3**

1. A uniform solid cylinder rolls smoothly along a horizontal floor, then up a ramp. What is the maximum height from the floor that the cylinder will reach if its initial center-of-mass velocity is 2 m/s?

   (1) 0.3 m (2) 1 m (3) 2.5 m (4) 4.2 m (5) 5.3 m

2. A uniform plank is supported by a hinge on a wall at one end and by a rope at its other end as shown in the figure. Rank the different cases according to the magnitude of the tension in the rope with the maximum first.

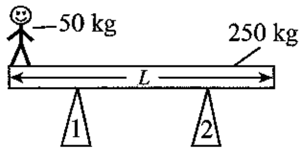
   

   (1) 1,3,2 (2) 2,1,3 (3) 2,3,1 (4) 3,2,1 (5) 3,1,2

3. A uniform ladder of mass 10 kg and length 2 m is placed on an icy smooth horizontal surface and leans against a rough wall with a static friction coefficient $\mu_s = 0.25$. Find the minimum (non-zero) angle θ, where θ is measured between the ladder and the ice, for which the ladder will be in balance.

   (1) 90 (2) 35 (3) 45 (4) 55 (5) 20

4. A gymnast with mass m = 50 kg stands on the end of a uniform balance beam as shown in the figure. The beam has a length L and a mass of 250 kg (excluding the supports). Each support is a distance $L/4$ from its end. Find the magnitude and the direction of the force on the beam due to support 2.

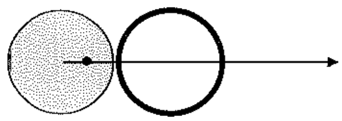
   

   (1) 1000 N, up (2) 1500 N, up (3) 1500 N, down (4) 3000 N, down (5) 500 N, down

5. The planet Jupiter is located approximately 5.2 AU (astronomical units) from the Sun. How long does it take Jupiter to orbit the Sun? (1 AU = the distance between the Earth and the Sun.)

   (1) 11.9 years (2) 140.6 years (3) 5.2 years (4) 1 year (5) 27 years

6. A uniform solid sphere is placed next to a uniform spherical shell (see figure). Both have the same mass M and radius R. What is the magnitude of the gravity force on a 1 kg particle placed on the axis x at a distance $x = R/2$ as shown on the diagram?

   (1) $(1/18)GM/R^2$ (2) $(17/18)GM/R^2$ (3) $(32/9)GM/R^2$ (4) $(40/9)GM/R^2$ (5) GM/R^2

7. Four particles each of mass m are placed at the corners of a square with an edge length L. One particle is moved from its position to "infinity". What is the work done by the net gravity force on this particle?

   (1) $-\frac{2.7Gm^2}{L}$ (2) $-\frac{3.0Gm^2}{L}$ (3) $-\frac{5.0Gm^2}{L^2}$ (4) $-\frac{3.0Gm^2}{L^2}$ (5) $-\frac{6.0Gm^2}{L^2}$

8. A man holding a weight in each hand stands at the center of a horizontal frictionless rotating turntable with his arms extended. The effect of the weights is to double the rotationial inertia of the system. As he is rotating, the man opens his hands and drops the two weights. They fall outside the turntable. Then:

(1) his angular velocity remains about the same.
(2) his angular velocity doubles.
(3) his angular velocity is halved.
(4) the direction of his angular momentum vector changes.
(5) his rotational kinetic energy increases.

9. To measure the mass of a planet with the same radius as Earth, an astronaut drops an object from rest (relative to the planet) from an altitude of one radius above the surface. When the object hits, its speed is 4 times what it would be if the same experiment were carried out on Earth. In units of Earth masses, the mass of the planet is:

(1) 16 (2) 2 (3) 4 (4) 8 (5) 32

10. A fir wood board floats in fresh water with 60% of its volume under water. The density of the wood in g/cm^3 is:

(1) 0.6 (2) 0.4 (3) 0.5 (4) less than 0.4 (5) more than 0.6

11. A main water line enters a house 2.0 m below ground. A smaller diameter pipe carries water to a faucet 5.0 m above ground, on the second floor. Water flows at 2.0 m/s in the main line and at 7.0 m/s on the second floor. If the pressure in the main line is 2.0×10^5 Pa, then the pressure on the second floor is:

(1) 1.1×10^5 Pa (2) 5.3×10^4 Pa (3) 1.5×10^5 Pa (4) 2.5×10^5 Pa (5) 3.4×10^5 Pa

12. A block attached to a spring oscillates in simple harmonic motion along the x axis. The limits of its motion are $x = 10$ cm and $x = 50$ cm, and it goes from one of these extremes to the other in 0.25 s. Its amplitude and frequency are:

(1) 20 cm, 2 Hz (2) 40 cm, 2 Hz (3) 40 cm, 4 Hz (4) 25 cm, 4 Hz (5) 20 cm, 4 Hz

13. A simple pendulum is suspended from the ceiling of an elevator. The elevator is accelerating upwards with acceleration a. The period of the pendulum, in terms of its length L, g, and a is:

(1) $2\pi\sqrt{L/(g+a)}$ (2) $2\pi\sqrt{L/g}$ (3) $2\pi\sqrt{L/(g-a)}$ (4) $2\pi\sqrt{L/a}$ (5) $(1/2\pi)\sqrt{g/L}$

14. A wave is described by $y(x,t) = 0.1 \sin(3x + 10t)$, where x is in meters, y is in centimeters, and t is in seconds. The wavelength is:

(1) $2\pi/3$ m (2) 6π m (3) 3π m (4) $\pi/3$ m (5) 0.1 cm

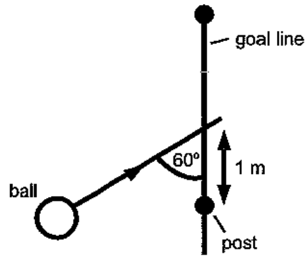
15. Water waves in the sea are observed to have a wavelength of 1000 ft and a frequency of 0.07 Hz. The velocity of these waves is:

(1) 70 ft/s (2) 0.0007 ft/s (3) 7 ft/s (4) 700 ft/s (5) none of these

16. The tension in a string with a linear density of 0.0010 kg/m is 0.40 N. A 100 Hz sinusoidal wave on this string has a wavelength of:

(1) 20 cm (2) 0.05 cm (3) 5.0 cm (4) 500 cm (5) 2000 cm

17. A soccer ball of mass 2 kg travels at 5 m/s at an angle of 60° to the goal line as shown, crossing the line 1 meter away from the goalpost. What is the magnitude of its angular momentum (in kg m^2/s) about the goalpost?



(1) $5\sqrt{3}$
(2) 5
(3) 10
(4) 0
(5) $10\sqrt{3}$

18. A piece of cork of density 200 kg/m^3 and volume 0.02 m^3 is placed under water (density 1000 kg/m^3) and released. What is the net force on the cork?

(1) 160 N (2) 200 N (3) 40 N (4) 240 N (5) 100 N

19. A merry-go-round of mass M and radius R can be considered as a uniform disk. It is rotating with $\omega = 0.6$ rad/s. A sack of sand, also of mass M, is dropped at a distance of $R/2$ from the axis and sticks to the merry-go-round. What does the angular velocity become now in rad/s?

(1) 0.4 (2) 0.2 (3) 0.3 (4) 0.6 (5) 0.1

20. A uniform spherical ball (mass = M, radius = R, $I = 2MR^2/5$ around its axis) rolls without slipping with velocity 5 m/s. With what velocity must a block of ice of the same mass M slide with no friction to have the same kinetic energy as the ball?

(1) 5.9 m/s (2) 5.0 m/s (3) 7.0 m/s (4) 3.6 m/s (5) 4.2 m/s

1. Conservation of energy.

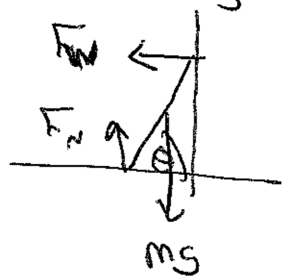
$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\times\frac{1}{2}\cancel{m}\cancel{r^2}\frac{v^2}{\cancel{r^2}} = \cancel{m}gh$$

$$\frac{3}{4}v^2 = gh \Rightarrow h = \frac{3}{4}\frac{v^2}{10} = \frac{12}{40} = 0.3\text{ m}$$

2. The ropes have each to give a torque equal in size to the torque due to the plank as measured around the pivot. $\tau = T \times r_\perp$ where $r_\perp$ is perpendicular distance between line of rope and pivot. $(r_\perp)_1 < (r_\perp)_3 < (r_\perp)_2$

So $T_1 > T_3 > T_2$

3. 

Only force is x direction is F_W.

Stability $\Rightarrow F_W = 0$

$\Rightarrow \theta = 90°$

4. Take torque around 1. Take force from pivot 2 to be positive up.

$$\frac{L}{4}\times 250\times g = \frac{L}{2}\times F_2 + \frac{L}{4}\times 50\times g$$

$$2500 = 2F_2 + 500$$

$$F_2 = +1000\text{ N}$$

5. From first principles $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$$T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T} \Rightarrow \frac{r^2}{rT^2} \propto \frac{1}{r^2} \Rightarrow r^3 \propto T^2$$

$$\Rightarrow \frac{T_2}{T_1} = \sqrt{5.2^3} = 11.9$$ (Kepler's 3rd Law)

$$\therefore F = \frac{GMm}{\left(\frac{3R}{2}\right)^2} - \frac{G(M/8)m}{(R/2)^2}$$

due to shell

due to mass of solid sphere – the part with $r < \frac{R}{2}$

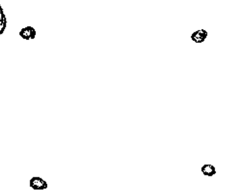
Left to right is positive

$m = 1$

$$= \frac{GM}{R^2}\left(\frac{4}{9} - \frac{1}{2}\right) = -\frac{1}{18}\frac{GM}{R^2}$$

7.

consider this particle



Moving a mass to infity means gravity is doing negative work

Change in energy $= -\frac{Gm^2}{L} - \frac{Gm^2}{L} - \frac{Gm^2}{\sqrt{2}L}$

$= -2.7\frac{Gm^2}{L}$

8. A weights are dropped there is no torque applied to man or weights, and their angular momentum stays the same. As the rotational inertias are also unchanged, this means the man + turntable keeps his angular velocity (note that weights do not fall vertically)

9. $-\left(\frac{GMm}{2R} - \frac{GMm}{R}\right) = \frac{1}{2}mv^2$

so $v^2 \propto M$ for constant R

10. Archimedes Principle.

$$\rho_w V_{WD}\, g = \rho_f V_f\, g \qquad \text{but} \quad \frac{V_{wd}}{V_f} = 0.6$$

volume water displaced

$\Rightarrow \rho_f = 0.6\,\rho_w = 0.6\ \mathrm{g/cm^3} \quad (600\ \mathrm{kg/m^3})$

11. $p + \frac{1}{2}\rho v^2 + \rho g y =$ a constant

$$P(\text{second floor}) = P_{\text{mains}} - \rho g(7.0) - \tfrac{1}{2}\rho 7^2 + \tfrac{1}{2}\rho 2^2$$

$$= 2.0\times 10^5 - 7\times 10^4 - 2.45\times 10^4 + 0.2\times 10^4$$

$$\approx 1.1\times 10^5\ \mathrm{Pa}$$

12. It travels 40 cm which is $2A \Rightarrow A = 20$ cm
It takes 0.25 s for a half-cycle $\Rightarrow \nu = 2$ Hz

13. Effectively this is like being on a planet where acceleration due to gravity is $g+a \Rightarrow T = 2\pi\sqrt{\frac{L}{g+a}}$

14. $y(x,t) = 0.1 \sin(3x + 10t)$

of the form $y = y_m \sin(kx - \omega t)$

$k = 3 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{3}$ meters

15. $v = \lambda f = 1000 \times 0.07 = 70$ ft/s

16. $v = \sqrt{\frac{\tau}{\mu}}$ where $\tau = 0.4$, $\mu = 0.001 \Rightarrow v = 20$ m/s

$v = \lambda f \Rightarrow 20 = \lambda 100 \Rightarrow \lambda = 0.2$ m (20 cm)

17. Angular momentum $= m(\vec{r} \times \vec{v}) = 2 \times 1 \times 5 \sin 60^\circ$

$= \frac{10\sqrt{3}}{2} = 5\sqrt{3}$ kg m²/s

18. Net force $= F_{up} - F_{Down} = \rho_w V_c g - \rho_c V_c g$

$= 1000 \times 0.02 \times 10 - 200 \times 0.02 \times 10$

$= 800 \times 0.2 = 160$ N $\uparrow$

19. Angular momentum conservation $I_i \omega_i = I_f \omega_f$

$\frac{1}{2} MR^2 \omega_i = \left[\frac{1}{2} MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega_f$

$\omega_f = \frac{2\omega_i}{3} = 4$ rad/s

20. $$\left(\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2\right)_{BALL} = \frac{1}{2}MV_{ICE}^2$$

$$\frac{1}{2}MV^2 + \frac{1}{2}\times\frac{2}{5}M\frac{V^2}{R^2}R^2 = \frac{1}{2}MV_{ICE}^2$$

$$\left(\frac{1}{2} + \frac{1}{5}\right)V^2 = \frac{1}{2}V_{ICE}^2$$

$$\Rightarrow \frac{V_{ICE}^2}{25} = \frac{7}{5} = V_{ICE} \sim 5.9\, m/s$$