PHY2048 Exam 1 Formula Sheet

Vectors

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ Magnitudes: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ $|\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$

Scalar Product: $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$ $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ (θ = angle between $\vec{a}$ and $\vec{b}$)

Vector Product: $\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$

Magnitude: $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$ (θ = smallest angle between $\vec{a}$ and $\vec{b}$)

Motion

Displacement: $\Delta x = x(t_2) - x(t_1)$ (1 dimension) $\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$ (3 dimensions)

Average Velocity: $v_{ave} = \dfrac{\Delta x}{\Delta t} = \dfrac{x(t_2) - x(t_1)}{t_2 - t_1}$ (1 dim) $\vec{v}_{ave} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$ (3 dim)

Average Speed: s_{ave} = (total distance)/Δt

Instantaneous Velocity: $v(t) = \dfrac{dx(t)}{dt}$ (1 dim) $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt}$ (3 dim)

Relative Velocity: $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$ (3 dim)

Average Acceleration: $a_{ave} = \dfrac{\Delta v}{\Delta t} = \dfrac{v(t_2) - v(t_1)}{t_2 - t_1}$ (1 dim) $\vec{a}_{ave} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$ (3 dim)

Instantaneous Acceleration: $a(t) = \dfrac{dv(t)}{dt} = \dfrac{d^2x(t)}{dt^2}$ (1 dim) $\vec{a}(t) = \dfrac{d\vec{v}(t)}{dt} = \dfrac{d^2\vec{r}(t)}{dt^2}$ (3 dim)

Equations of Motion (Constant Acceleration)

$v_x(t) = v_{x0} + a_x t$ $v_y(t) = v_{y0} + a_y t$ $v_z(t) = v_{z0} + a_z t$

$x(t) = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2$ $y(t) = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2$ $z(t) = z_0 + v_{z0}t + \tfrac{1}{2}a_z t^2$

$v_x^2(t) = v_{x0}^2 + 2a_x(x(t) - x_0)$ $v_y^2(t) = v_{y0}^2 + 2a_y(y(t) - y_0)$ $v_z^2(t) = v_{z0}^2 + 2a_z(z(t) - z_0)$

Newton's Law and Weight

$\vec{F}_{net} = m\vec{a}$ (m = mass) Weight (near the surface of the Earth) = W = mg (use g = 9.8 m/s^2)

Magnitude of the Frictional Force

(μ_s = static coefficient of friction, μ_k = kinetic coefficient of friction)

Static: $(f_s)_{\max} = \mu_s F_N$ Kinetic: $f_k = \mu_k F_N$ (F_N is the magnitude of the normal force)

Uniform Circular Motion (Radius R, Tangential Speed v = Rω, Angular Velocity ω)

Centripetal Acceleration & Force: $a = \dfrac{v^2}{R} = R\omega^2$ $F = \dfrac{mv^2}{R} = mR\omega^2$ Period: $T = \dfrac{2\pi R}{v} = \dfrac{2\pi}{\omega}$

Projectile Motion

(horizontal surface near Earth, v_0 = initial speed, θ_0 = initial angle with horizontal)

Range: $R = \dfrac{v_0^2\sin(2\theta_0)}{g}$ Max Height: $H = \dfrac{v_0^2\sin^2\theta_0}{2g}$ Time (of flight): $t_f = \dfrac{2v_0\sin\theta_0}{g}$

Quadratic Formula

If: $ax^2 + bx + c = 0$ Then: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$