PHY2048 Exam 2 Formula Sheet

Work (W), Mechanical Energy (E), Kinetic Energy (KE), Potential Energy (U)

Kinetic Energy: $KE = \frac{1}{2}mv^2$ Work: $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \xrightarrow{\text{Constant } \vec{F}} \vec{F} \cdot \vec{d}$ Power: $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

Work-Energy Theorem: $KE_f = KE_i + W$ Potential Energy: $\Delta U = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$ $F_x(x) = -\frac{dU(x)}{dx}$

Work-Energy: W(external) = ΔKE + ΔU + ΔE(thermal) + ΔE(internal) Work: W = -ΔU

Gravity Near the Surface of the Earth (y-axis up): $F_y = -mg$ $U(y) = mgy$

Spring Force: $F_x(x) = -kx$ $U(x) = \frac{1}{2}kx^2$

Mechanical Energy: $E = KE + U$ Isolated and Conservative System: $\Delta E = \Delta KE + \Delta U = 0$ $E_f = E_i$

Linear Momentum, Angular Momentum, Torque

Linear Momentum: $\vec{p} = m\vec{v}$ $\vec{F} = \frac{d\vec{p}}{dt}$ Kinetic Energy: $KE = \frac{p^2}{2m}$ Impulse: $\vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$

Center of Mass (COM): $M_{tot} = \sum_{i=1}^{N} m_i$ $\vec{r}_{COM} = \frac{1}{M_{tot}}\sum_{i=1}^{N} m_i\vec{r}_i$ $\vec{v}_{COM} = \frac{1}{M_{tot}}\sum_{i=1}^{N} \vec{p}_i$

Net Force: $\vec{F}_{net} = \frac{d\vec{P}_{tot}}{dt} = M_{tot}\vec{a}_{COM}$ $\vec{P}_{tot} = M_{tot}\vec{v}_{COM} = \sum_{i=1}^{N} \vec{p}_i$

Moment of Inertia: $I = \sum_{i=1}^{N} m_i r_i^2$ (discrete) $I = \int r^2 dm$ (uniform) Parallel Axis: $I = I_{COM} + Mh^2$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$ Torque: $\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$ Work: $W = \int_{\theta_i}^{\theta_f} \tau d\theta$

Conservation of Linear Momentum: if $\vec{F}_{net} = \frac{d\vec{p}}{dt} = 0$ then $\vec{p}$ = constant and $\vec{p}_f = \vec{p}_i$

Conservation of Angular Momentum: if $\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$ then $\vec{L}$ = constant and $\vec{L}_f = \vec{L}_i$

Rotational Varables

Angular Position: $\theta(t)$ Angular Velocity: $\omega(t) = \frac{d\theta(t)}{dt}$ Angular Acceleration: $\alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2}$

Torque: $\tau_{net} = I\alpha$ Angular Momentum: $L = I\omega$ Kinetic Energy: $E_{rot} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$ Power: $P = \tau\omega$

Arc Length: $s = R\theta$ Tangential Speed: $v = R\omega$ Tangential Acceleration: $a = R\alpha$

Rolling Without Slipping: $x_{COM} = R\theta$ $v_{COM} = R\omega$ $a_{COM} = R\alpha$ $KE = \frac{1}{2}Mv_{COM}^2 + \frac{1}{2}I_{COM}\omega^2$

Rotational Equations of Motion (Constant Angular Acceleration α)

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2(t) = \omega_0^2 + 2\alpha(\theta(t) - \theta_0)$$