1. There are $(36)^3$ cubic inches in a cubic yard. There are $(2.54)^3$ cubic centimeters (that is cm^3) in a cubic inch. So, we have $(2.54)^3 x(36)^3 = 764554.857984$

(The 2.54 is defined as the conversion from centimeters to inches, so the above is rather exact, but rarely are the answers to exam questions to more than 2 or 3 significant figures).

This question had the highest success rate.

2. Average velocity is the net displacement divided by the total time. The beginning and end points are 50 km apart (it's a 3,4,5 triangle), so the displacement has a magnitude of 50 km. It takes 1 hour for the first leg and one hour for the second, so that means it took a total of 2 hours. The average velocity is thus 50/2 = 25 km/hr

   A surprising number of people did not divide by 2 and thus got 50 km/hr

3. We differentiate to get $\mathbf{v}(t) = 40t\mathbf{i} + 15t^2\mathbf{j}$ noting that we are differentiating the two coordinates separately. Then differentiate again to get $\mathbf{a}(t) = 40\mathbf{i} + 30t\mathbf{j}$. Then set t=2 and you get $\mathbf{a}=40\mathbf{i}+60\mathbf{j}$. Then use the pythagorean formula to find $|\mathbf{a}|$

4. The horizontal component of its speed is a constant. The vertical component gets smaller, is momentarily zero, then rises again. Clearly the moment that the total speed is least when it is at the top of its trajectory.

   To calculate how high it gets, we simply use the vertical components - it starts off at:

$$v_y = V\sin(\theta) = 40 \cdot 0.5 = 20 \left(\frac{m}{s}\right)$$

   So it reaches the top at a time given by $\mathbf{v}=\mathbf{v}_i+\mathbf{a}t$

$$0 = v_y = 20 - 9.8 \cdot t$$

   Where we have used "up is positive" and so the acceleration due to gravity has a minus sign. This gives t=2.04 s. The distance, y, at this time, will thus be given by: $\mathbf{d}=\mathbf{v_i}t + 0.5\mathbf{a}t^2$

   and so: $y = 20*2.04 - 0.5*9.8*2.04^2 = 20.4$m

5. Consider the swim with no current. You would go 120 meters across and 50 meters downstream. By the pythagoream theorem that makes 130 meters. You swim at 2.6 m/s, so that is 50 seconds. Very convenient numbers. That is the vector of what you do in still water. Now you add the vector of the water going downstream. The time taken does not change - the added vector only changes the distance traveled downstream by a further 50 meters. 50 meters in 50 seconds tells us that the velocity of the flow is 1 m/s.

This question was surprising poorly one despite there being a more difficult one on the same subject in the homework.

6. As the velocity is constant, the net force on the block has to be zero. The value of the velocity (and the mass) are then not important. The y-components of the two forces (as shown) cancel. The x components are each $F\cos(60^0) = 1N$, so that they add to give a force in the positive x-direction of 2N. Therefore they have to be "balanced" by a force in the -x direction of 2N.

   We've had several "Newton's first law" questions in class. Constant v means no net force.

7. This simply Newton's Third Law. The smaller pushes on the bigger one with a force F, the bigger one pushes the smaller one with the same magnitude of force.

   The scores on this one were disappointing, and I suspect people were making it more complicated than it was. The smaller car clearly has another force on it (there is an engine making the wheels turn and thus pushing the car forward), but that is not the question.

8. Lets call down positive. Then there will be no negative numbers. We will call t=0 as the time of the release of the first ball. So the distance dropped by the first is

   $d_1 = v_{1i}t + 0.5gt^2 = 0.5gt^2$

   Distance fallen by the second = $d_2 = v_{2i}(t-1) + 0.5g(t-1)^2$ [where $v_{2i} = 20$ m/s]

   These are the same, so $0.5gt^2 = 20t - 20 + 0.5gt^2 - gt + 0.5g$

   $20 = t(20-g) + 0.5\ g$

   $15.1 = 20t - 9.8t = 10.2t$

   $t = 15.1/10.2 = 1.48$ seconds

   So $d_1 = 0.5x9.8x1.48^2 = 10.7$ meters

   Check: $d_2 = 20x0.48 + 0.5x9.8x(0.48)^2 = 10.7$ meters

9. The cross product of vectors in the x and y directions, gives a vector in the z direction. The dot product of a vector in the x direction with one in the z direction is 0 (because they are at right angles to each other). Therefore the answer is zero.

   Most people got this.

10. The time taken to hit the ground is independent of its horizontal motion. Using the usual kinematic equations for constant acceleration (that due to gravity), we can find this out by 200 m = (0.5 x 9.8 x t^2)m and so it takes 6.39 seconds to hit the ground. The vertical component of its velocity when it hits is $v_y = v_{iy} + gt = 62.6$ m/s (taking down to be positive). In the horizontal direction, the component of the velocity is a CONSTANT 200 m/s. So, to find the speed on impact, we need to take the two components in quadrature , so $\text{speed}^2 = 200^2 + 62.6^2$ and so speed = 210 m/s

    Some people forgot that the horizontal component of velocity is not affected by the downward one – as we demonstrated in class.

11. We define the x-direction as the initial direction, A->B. There is an abrupt change of direction as the kick (short and sharp) imparts a component of velocity in the y-direction. After that, there are no forces on the puck so it goes in a straight line diagonally in the x-y plane (i.e. answer B)

    Note that after the kick, there is NO force (in the plane of the ice), therefore it MUST be a straight line. Newton's first law rules!

12. Because the scale is reading low, it tells us that the acceleration is down. There is no information on the velocity.

    We can deduce the above from Newton's Laws. The scale senses the normal force (and converts to kg). So the normal force is 40*g* (up), the weight is 50*g* (down) and so the net force is 10*g* (down). The force and the acceleration are in the same direction and is thus down. There is no information on the velocity as the laws of physics are the same in all inertial rest frames (a restatement of Newton's First Law).

13. T-mg=ma (force balancing by using a free-body diagram on the monkey).

    Therefore a=(T-mg)/m, where we have defined up as positive.

Now for the distance traveled starting from rest and with constant acceleration.

$d=0.5at^2$ and we can plug in a from above and rearrange.

Therefore: $t^2=2md/(T-mg)$

Plug in m, d, T(max), g to get the answer.

14. If the tension in the string is T, then the vertical component of it is $T\sin(37^0)$. As there is no acceleration and thus no net force vertically, in the vertical direction; $T\sin(37^0)=mg$

The horizontal component of the Tension does give an acceleration (Newton's Second Law); $T\cos(37^0)= mv^2/r$, where r is the radius of the circle that the mass is moving in. $r = R\cos(37^0)$, where R is the length of the string. This makes: $v^2=T\cos(37^0)*R\cos(37^0)/m$

substituting in $T=mg/\sin(37^0)$

$v^2 = \cos^2(37^0)*g*R/\sin(37^0)$

I thought this was one of the more demanding questions, and so it proved.

15. **A**-**B** = 2**i** + 0**j** -2**k**. Then by the pythagorean theorem, the magnitude is given by $|\mathbf{A}\text{-}\mathbf{B}|^2 = 2^2 + 2^2$ and |**A**-**B**| = 2.82

16. Newton's second law is F=ma. In the vertical direction, the man experiences two forces, the Normal Force (which we are trying to find) and the Gravitational Force. They act (in opposite directions) to give the net force which is ma, where a is the acceleration (which is down) and is $a=v^2/R$ because it is going in a circle of radius R and it is going at a constant speed of v.

So, with down being positive, $mg - F_N = mv^2/R$

$50*9.8 - F_N = 50*20^2/50$

$F_N = 490 - 400 = 90N$

We did something very similar in class.

17. The block is stationary and so has no net force on it. If we look at components along the slope, there is a force down the slope, due to gravity, of $mg \sin(20^0) = 16.8$ N There must be an equal and opposite force up the slope due to friction, so f = 16.8 N.

Note that Amonton's Laws do not provide the answer. For static friction:

$f \le \mu_s F_N$ and we know that $F_N = mgcos(\theta)$

That tells us in this case that $f \le 27.6$ Newtons. If the value of f we found from the calculation were above 27.6 Newtons, then the question would not make sense. But the inequality does not tell us what the value of f really is.

Note that it was stressed in class that the friction force of a block on a horizontal plane which has nothing pushing it horizontally is ZERO. 20 degrees is a small angle, it does not take much friction to keep it stationary to overcome the small gravitational force acting down the slope.
18. There is only one value of the acceleration, though the acceleration of B is down and of A is horizontal

Let's look at free-body diagrams for A and B separately. For block B

$$M_B g - T = M_B a$$

where we have taken down as being positive.

For the block A, we have:

$$T - \mu_k M_A g = M_A a$$

These are two equations, with two unknowns (T and a), and there are various ways of solving. One way is to solve for "a" first by adding the two equations.

$$(M_B - \mu_k M_A)\, g \;=\; (M_A + M_B)\, a$$

and putting numbers in yields:

$$a = \left(\frac{(2 - 1.5)}{2 + 3}\right) g$$

and so a = 0.98 m/s^2

This seems reasonable, it will not accelerate much because the force of friction on the top block is not so different from the force of gravity on block B.

But the question asks for T, and using the first equation we have

$$2 \times 9.8 \;- T \;= 2 \times 0.98$$

$$T \;= 17.6N$$

and just to check, we can put it into the second equation:

$$T - 0.5 \times 3 \times 9.8 \;=\; 3 \times 0.98$$

= 17.6 Newtons

19. We cannot use the familiar kinematic equations because the acceleration is not a constant. Therefore we can integrate.

$$v = \int (6t + 1)\, dt \;=\; \frac{6}{2} t^2 + t \;+\; v_0$$

As it starts at rest, we know that $v_0 = 0$

And we can integrate the velocity to find the displacement.

$$\Delta r = \int_0^2 \left(3t^2 \;+ t\right) dt \;= 8 + 2 \;=\; 10$$

This is *displacement*, not necessarily the same as the distance traveled. However, in this case, as there is no changing of the direction, the distance traveled and the displacement are the same size.

20. Keeping the "up is positive" notation, we have the position of the stone is given by

$\mathbf{y} = \mathbf{y}_i + \mathbf{v_i}t + 0.5\mathbf{a}t^2$ where y_i is the initial distance off the ground which will be our answer, and we know that y=0 at t = 3s. Therefore

$-y_i = 2 \times 3 - 0.5 \times 9.8 \times 3^2 = -38.1$ meters

Therefore the balloon is 38.1 meters off the ground.

It is important to note that the only acceleration that we need to take into account is that due to gravity. The acceleration of the balloon at the moment of release is not important. All accelerations come from forces, and the only force on the stone AFTER the release is that of gravity.