1. Here we can imagine the icecube sliding down (and getting faster), and turning. At the bottom, it reaches a maximum speed (so that, at that moment, it is not getting faster or slower), but it IS moving in a circle and is thus accelerating towards the center of the circle (upwards at that moment). If we can find that maximum speed we can find the acceleration.
   Finding the maximum speed is best performed by energy conservation. The total distance moved *in the direction of the force,* is mgR, so that is equal to the total kinetic energy at the lowest point. Therefore:
   $mgR=0.5mv^2$ and the mass cancels. So therefore $v^2=2gR$. That is the speed at the bottom. Now for an object going in a circle, the acceleration is given by v^2/R (there is only one radius in this question, so the R is the same R as used above). Therefore a = 2g.
   (Note that the last answer is clearly wrong because it is dimensionally inconsistent)

2. Although in principle this could be done using Newton's Laws of Motion, it would be very difficult to do it that way because for a spring, the force depends on position (F=-kx). However, energy conservation makes the problem much easier. We have a conservative system, so that total energy is conserved, and what we are seeing is the loss of potential energy making a gain in the kinetic energy. The potential energy of a spring is $0.5kx^2$ and we take care to convert to meters to be consistent.
   $\Delta U + \Delta K = 0$
   $0.5k(x_f^2-x_i^2) + 0.5mv^2 = 0$
   $0.5*200[(-0.1)^2-(0.05)^2] = 0.5*(0.5)v^2$
   $100*(0.01 – 0.0025) = 0.25v^2$
   $0.75 = 0.25\ v^2$
   v=1.7 m/s

   Note that the answer would have been the same had the question asked for the speed at x = -5cm, because the potential energy in the final position is the same in the two cases.

3. For a conservative force, we know that the potential energy difference is given by:
   $$\text{Potential Energy: } \Delta U = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}\ .$$
   As the force is constant (it doesn't depend on position), this simplifies to
   ΔU = -**F.d** where **F** is force, and **d** is the total displacement. As this is one dimensional, it makes it even easier, and it just Force x Distance = - (change in potential energy).
   Note that I could have surmised that the change in potential energy is negative, by noting that an object would, generally speaking, *like* to go from the origin the a place in the positive x-axis, so that that location is clearly a position of lower potential energy.

4. The work done by gravity is **F.d** and as the force is DOWN, we need to consider only the up-down coordinate of the displacement (left-right is not important). As we all know, the sin of 53^0 is 0.8, that distance moved up is 4 meters, and so work done by gravity is mg(-4) = -2*9.8*4 = -79 Joules. Note that if an object moves up, the work done by gravity is clearly negative, which reduces the options of the answer.

5. If there was no friction, we could find the speed at the bottom by using conservation of energy. However, we are told the speed (and therefore the kinetic energy at bottom), and we find it is much less than the potential energy lost. The difference is because of the energy lost to the mechanical system because of the frictional force. If we find this difference, and we know the distance that friction acted, we can find the frictional force.
   So, the change in mechanical energy = $\Delta U + \Delta K = -mgh + 0.5mv^2 = -60*9.8*5 + 0.5*60*2^2 = -2820$J. So the work done by friction is -2820J. It acts over 5 meters of length, and so the frictional force is 2820/5 = 564J (note that the motion acts in the opposite direction to the force of friction and hence the negative sign).

6. The x and y coordinates can be worked out separately. The two plates have the same mass (call it m), and we can consider the center of mass of each of them to be at their centers, that gives (with everything in appropriate units, the distances being in cm):
   $x_{com} = (m*0.5 + m*2.5)/(m+m) = 1.5$, and $y_{com} = (m*1.5 + m*0.5)/(m+m) = 1.0$
   The distance from the origin is given by $r^2 = x_{com}^2 + y_{com}^2$

7. Power is the rate of doing work. With a constant velocity and constant force, then Power = **F.v**
   As the force and velocity in the same direction, that makes Power = Fv. The net force on the box is zero (Newton's First Law), so therefore the force supplied by the motor is equal (and opposite) to the force of friction. Using the laws of friction, Power = (0.4*5*9.8)*2 = 39.2 Watts

8. It is always a good idea to draw the potential energy diagram. Clearly there is a minimum of potential energy given at the time that dU/dx = 0, and this is an x value given by -4x-4 = 0 and thus x=-1. An object will move from the origin, to that position (when its speed is a maximum), and then continue till U=0 again, and then oscillate. It will actually be rather similar to a bowling ball on a string. To find when U(x) = 0, we solve the (very simple) quadratic equation 2(x+2)x = 0, and thus x-=2 is the second case that corresponds to U(x) = 0. Thus the particle goes as far as x=-2 and then oscillates between x=0 and x=-2.

9. There are many ways of doing this problem (for instance, Newton's 2nd and 3rd laws together with the kinematic equations). However, the easiest way is to note that there are no external forces, and so the center-of-mass of the system must be constant. If we work out the center-of-mass position originally, using Jill's position as being the origin (x=0), then that original center-of-mass position is the same as the final center-of-mass position which is where Jack and Jill meet.

   $x_{COM} = (m_1x_1 + m_2x_2)(m_1 + m_2) = (60*0 + 90*20)/(150) = 12$ meters, so Jill moves 12 meters.

10. This question has no external forces. We know that momentum is conserved. So therefore:

    $|M_1v_1| = |M_2v_2|$ and so $|v_2| = |v_1|M_1/M_2$ (momentum conservation)

We know that kinetic energy is $0.5Mv^2$

So, $E_1/E_2 = (0.5M_1v_1^2)/(0.5M_2v_2^2)$

Now we can substitute in the relationship from momentum conservation, and we get

$E_1/E_2 = M_2/M_1 = 3$

The more massive the object, the less kinetic energy it has in an elastic collision. When you jump up in air, momentum is conserved. The earth is so massive, the energy it gets is very, very, small, and virtually all the energy expended goes into you, not the earth.

11. This collision conserves momentum (all collisions do), but does not conserve kinetic energy (it is clearly inelastic – hence the "imbedded in the wood"). The block is free to move afterwards, so there will be some kinetic energy in the (wood+bullet) afterwards.

Beforehand, the energy of the bullet in SI units is: $E = 0.5*(.005)*200*200 = 100$ J

Momentum conservation tells us that after the collision the velocity is $v = (5/25)*200 = 40$ m/s

So afterwards, the kinetic energy of the (wood+bullet) is $0.5*(.025)*40*40 = 20$ J

So (100 – 20) = 80 Joules is "lost" to the kinetic system.

12. This question is best solved using the concept of impulse. Over a period of time, t.

$J = F_{ave}\, t = \Delta(mv)$

We must remember that impulse (and momentum) are vector quantities. Thus, taking UP to positive,

$F_{ave}\, t = m(v_f-v_i) = 0.06*(v_f - v_i)$ noting that v_i is negative.

To find the two speeds, you can use the kinematic equation for dropping at rest that:

$v^2 = 2gy$, where x is the distance. (This is the same as energy conservation: $mgy = 0.5mv^2$)

$v_f = 4.4$ m/s, and $v_i = 6.2$ m/s, so $F_{ave} = 0.06*(10.6)/0.02 = 32$ Newtons

13. The point's rotation is getting faster, and it is also moving in a circle. Therefore its linear acceleration is the sum in quadrature of its tangential acceleration and its radial acceleration.

To find the angular acceleration, we differentiate the angular velocity and get

α(at t=2second) = $3*0.5*2^2= 6$ rad/s^2 and so the tangential acceleration is $a_T = 0.5\alpha = 3$ m/s^2

At t =2 seconds, the angular velocity is $\omega = 8-0.5t^3 = 4$ rad/s, and so the radial acceleration is:

$a_R = r\,\omega^2 = 0.5*4^2 = 8$ m/s^2

We add these together as two vector components at right angles, to get the total linear acceleration:

$a = \sqrt{(8^2 + 3^2)} = 8.5\ m/s^2$

The fact that something moving at varying speed in a circle has TWO components of linear velocity, which should then be added (by using the square root of the sum of the squares) was something that was stressed in the lecture of October 17th.

14. The rotational inertia total is the sum of the rotational inertia of the components. The rod which is along the axis of rotation has NO rotational inertial The stick at right angles contributes $(1/3)ML^2$ (see formula sheet). For the hoop, we note that formula for an axis along a diameter, is $(1/2)ML^2$, but added to this we need an extra $M(2L)^2$ from the parallel axis theorem. This adds to $[(1/3)+(1/2)+2]ML^2 = (29/6)ML^2$.

15. We look at the free-body diagram for the box, taking down to be positive (which is the direction it will move).

$Mg - T = Ma$. But the acceleration of the box is related to the angular acceleration by $a = R\alpha$

Now, look at the tension in the string pulling the pulley. Torque = $TR = I\alpha$, where the Tension, T is the same tension as before.

$TR = I\alpha$ which gives $T= I\alpha/R$

Adding the two together equations together gets rid of T.

$Mg = MR\alpha + I\alpha/R$ and so $\alpha = Mg/(MR + I/R) = 5*9.8/(5*3 + 15/3) = 2.45\ rad/s^2$

We then multiply by 5 seconds to get the angular velocity.

16. This is clearly an energy conservation question. In all cases (as the masses are the same), the potential energy gained is proportional to the how far they climb before coming to a standstill. So, the question comes down to which one has the greatest kinetic energy? They all have the same center-of-mass velocity, so the kinetic energy associated with the motion of the center of mass is the same for all of them. However, the KE which is due to the rotation of the mass around its center of mass, which depends on the speed of rotation (the same for them all) and also the rotational inertia. The rotational inertia is greatest for the thin annular ring. This can be checked by looking at the cover sheet of the exams.

17. The basic physics principle here is angular momentum conservation. (Note that the collision is inelastic, and does not conserve energy).

Beforehand there is the angular momentum of the mud, which **L** = **r** x **p .** The angle is easy (right angle between r and p), so that makes L = 1.0 * 0.5 * 24 = 12 kgm^2/s (where the 1.0 is the distance to the center of the door, and the 0.5 is the mass of the mud).

That must be the angular momentum afterwards which is the sum of that due to the mud and that due to the door. The mud part is $MR^2\omega = 0.5\omega$, whereas the door part is "I" which is what we are asked to find.

$L = (I + 0.125)\omega$, so: $12 = (I + 0.5)*3$ and so: $I = 3.5\ kg.m^2$

18. Again, an inelastic "coming together" of two moving objects, so kinetic energy is not expected to be conserved. Angular momentum conservation tells us that the total angular momentum must be conserved. In terms of the rotational inertia, I, of one of the disks, the total angular momentum is $(4-1)I\omega = 2I\omega_f$. So when they go at the same angular speed, that makes the final angular velocity, $I\omega_f = 3I\omega$ and so $\omega_f = 1.5\ \omega$

19. This is a torque question. Mobiles will turn at their pivot points when they are not balanced. Being balanced means the net torque around these potential pivot points is zero. Work from the bottom, knowing that the torque around each pivot point has to add (counter-clockwise being positive + clockwise being negative) to zero. Taking into account the distances, the bottom left smiley face is thus clearly of mass A/2 = 1.0 kg. The right-most smiley face is thus 3*2 = 6 kg, so that means that the object B is 0.5*(6+3) = 4.5 kg

There was a very similar question to this in class on October 31st.

20. We know for a statics problem that the not only do the forces in all directions total to zero, so do the torques around all possible axes of rotation. If we take the axis of rotation where rope A holds the beam, we can solve in one step, noting that the mass of the beam acts as if it is concentrated at the half-way point.

So: Clockwise torque = counter-clockwise torque.

2*0.2*g + 5*0.5*g = 1.0*T

T = 28.4 Newtons