1. As we did in class, if there are no external forces, the center-of-mass of the system has to remain in the same location. So, if we consider the system to be boat+dog, we can set up an equation with the two sides being the center-of-mass before and after the walk. The mass of the boat itself is taken as being at the center of the boat - which therefore starts at 4 meters from the shore. The location of the dog from the shore (the answer) is given by X.

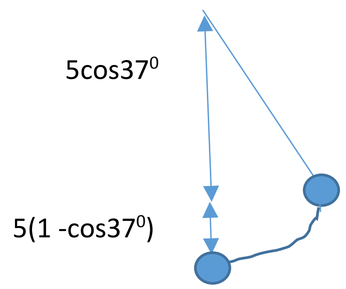
   (6.0 x 2.0) + (4.0 x 12.0) = 2.0X + 12.0(X+2.0)

   12 + 48 = 14X + 24 and so X = 36/14 = 2.6 meters

2. Mechanical energy conservation. The vertical distance that the ball falls is $L(1-\cos(37^0)) = L/5$ meters. Then we use $mgh = 0.5mv^2$ where the velocity is the velocity at the lowest point.

   v=sqrt(2.*9.8*L/5) = ?

   Now wait a minute, the question did not give the length. This was a mistake of our exam program – the text draft had the information but the computer cut it off. *Therefore there is no way to get an answer!* It was meant to say 5 meters (which some people guessed), which gives v =4.4 m/s. A few (very few) asked about this in the exam, but only after an hour or more by which time it was too late to let all the students know (some had left). The bottom line is the EVERYONE gets marked correct for this question. We are very sorry if this got people confused.



3. This is very similar to the H-iTT question in class (which was poorly done…) The rotational inertia of a stick around its midpoint is $(1/12)ML^2$ Now we have a shortened stick. Its mass is (3/4) the mass of the longer one, and its length is (3/4) the length of the longer one. So, therefore $I_2 = (3/4)x(3/4)^2$ times the original rotational inertia, so $I_2 = 0.42\ I_1$

4. In the same way as we had in class (which involved a hand grenade thrown horizontally, which is a harder problem), the center-of-mass of the trajectory of the object is not changed by the fact that it is split into two. So, we can find the center of mass at the t=2 seconds. As the only motion of the center of mass is UP, we use y as

variable, y=0 at ground level. $y=y_0+v_0t+0.5at^2$ = 15x2 – 0.5*9.8*2^2 = 30-19.6 = 10.4 meters. So we know that the center of mass is 10.4 meters off the ground at that moment. One half is on the ground, so the other half must be 20.8 meters off the ground. (n.b. we do not know where they are in the x direction, nor did we need to know when or where the explosion is).

5. The force is given by F= -dU/dx. So F(x) = 2x-3. The point mass starts off going in the positive x-direction, and at x=1 meter, it is given by F(x) = 1 Newton.

6. We know that the impulse is the change in momentum, and is given (for constant force) as the product of force and time. So the impulse is 50 x 0.1 = 5.0 kg m/s. The speed with which it hits the ground is given by $0.5mv^2$ = mgh and so v=sqrt(2*9.8*2) = 6.3 m/s, and so its momentum is mv = 3.15 kg m/s (downwards). This then changes to (5.0-3.15) kg m/s = 1.85 kg m/s upwards – remember that momentum is a vector! Thus the initial velocity up is 1.85/0.5 = 3.7 m/s. It then goes up a distance y given by $0.5m3.7^2$ = mgy and so y=0.5*3.7^2/9.8 = 0.7 meters.

7. As was stressed in class, when moving faster in a circle, there are two components of acceleration, one tangential and one radial (centripetal). The first one is given by Rα = 0.5*0.3 = 0.15 m/s^2 and the second is $R\omega^2$ = 0.5*$(2*0.3)^2$ = 0.18 m/s^2 these two orthogonal components must be combined to give the total a = sqrt(0.18^2 + 0.15^2) = 0.23 m/s^2

8. As we showed in class, you need to set up two free-body diagrams, one for the block of mass m, and the other for the pulley, and then relate them. The tension, T, is common to the two.

   For the mass: mg – T = ma
   For the pulley TR = Iα
   As the block accelerates down the pulley has angular acceleration such that a=Rα
   We also know the value of the rotational inertia of a uniform disk of mass M is given by $0.5MR^2$

   That makes the equation for the pulley: TR = $0.5MR^2$(a/R) and the radius cancels completely, giving T=0.5Ma and a = 2T/M
   Substituting into the first equation, we have
   mg-T=m2T/M
   Mmg = T(M+2m)
   T = Mmg/(M+2m)

   Note that the two answers with a negative sign on the bottom line are clearly impossible, as the Tension could be infinite.

9. The work done is the force times the distance. As there is no change in kinetic energy, the work done by the dog team must be equal in magnitude to the work done by the frictional force. The force of friction is the normal force times the coefficient of friction, and here the normal force is simply 5000 N as there are no other forces with vertical components.

   So the work done by friction is -5000*0.05*1000 = -2.5×10^5 and the work done by the dog team equal and opposite to this. (Yes, we know that dog teams often pull things called *sleds,* and some people use the word *sledges,* but we hoped everyone would know *sleigh* – if any time you don't understand an English word in an exam, please don't hesitate to ask.)

10. As it is going at constant speed, then the kinetic energy is not changed. Therefore the potential energy lost must be equal to the thermal energy increased (that is "dissipated by friction"). That is simply Mgh. You may wonder where the angle and coefficient of friction go to. They simply cancel out (there is only one coefficient of friction that works for one angle, but as we are told the speed is constant they disappear and it becomes really easier than it looks).

11. The work done is the integral of the dot product of force and displacement, $W = \int \boldsymbol{F}.\boldsymbol{dr}$ Here the displacement is entirely in the x direction, which makes life easy, as we only have to consider the x component of the force. So we have $W = \int_2^4 2xdx$ = 16 – 4 = 12J

12. This is energy conservation. The energy stored in the spring is $0.5kx^2$ and the mechanical energy turned into heat by friction is $F_N\mu_kD$ where D is the distance moved (0.4 meters), and F_N is the normal force. As there are no vertical forces other than gravity and so the normal force, F_N = 2.0*9.8 = 19.6N. Therefore we can set up an equation:

    $0.5 \times 200 \times 0.1^2 = 19.6 \times 0.4 \times \mu_k$ and so $\mu_k = 0.13$

13. A: As the motor is lifting the block at constant speed, the upward force of the cable is equal, and opposite, to the gravitational force on the block. It is thus constant.

    B: The block has constant speed and so it has no acceleration

    C: The net work done on an object is the change in its kinetic energy, and there is none. The motor does positive work, but gravity does negative work, and they cancel.

    D. **The mechanical energy of the Earth-Block system has increased, as the potential energy has increased.**

    E. As the velocity and mass of the block are constant, the KE must be a constant.

14. When the particle is released it will move faster for a while, hit a local maximum at b, slow down but not to a stop, hit a local minimum speed at c, speed up till d, then slow down coming momentarily at rest at e. It then changes direction and reverses the cycle to point a, and then starts all over again; at a it is again momentarily at rest before starting off in the positive direction again. Therefore:

    A is correct

    B is correct

    C. The particle never stops moving, so this is incorrect

    D. is not correct – it does not change direction at c, it simply changes from slowing down to speeding up

    E. At point d the rate of change of velocity (that is, acceleration) is indeed momentarily zero (as it is also at points b and c)

15. This is energy conservation, taking care to take into account the energy associated with the turning of the disk.

    KE beforehand = change in the potential energy

    $0.5mv^2 + 0.5I\omega^2 = mgh$

    The uniform disk has $I = 0.5mR^2$ around its axis of symmetry. Furthermore, rolling without slipping makes $v=R\omega$

    $0.5mv^2 + 0.5 \times 0.5R^2mv^2/R^2 = mgh$

    $0.75v^2 = gh$ and $h=3v^2/(4g)$

16. Here we use angular momentum conservation around the center of the stick. Beforehand the angular momentum is given by **r x p** and this is equal to (L/2)mv of the particle. Afterwards the particle is at rest (it is not obvious that this will happen, but it can, and it is stated to do so in the question), so the only angular momentum afterwards is that of the stick and that is $I\omega$, and as the stick will rotate around its center of mass, this is $(1/12)ML^2\omega$

    So we have $(L/2) \times (M/18) \times v = (1/12) \times M \times L^2\omega$

    $v/36 = L\omega/12$ and so $\omega=v/3L$

17. This is a classic angular momentum conservation question. The angular momentum is conserved, so taking the final angular velocity as ω_2 we have

    $I_1 \omega_1 = I_2 \omega_2$

    Beforehand the rotational inertia is: $I_1 = MR^2 + 0.5(4M)R^2 = 3MR^2$

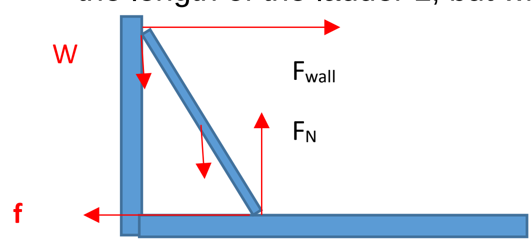
    Afterwards it is:

    $I_2 = M(R/2)^2 + 0.5(4M)R^2 = 2.25MR^2$ and so $I_2 = (3/4)I_1$

    So, $\omega_2 = (3/2.25) = (4/3)\ \omega_1$

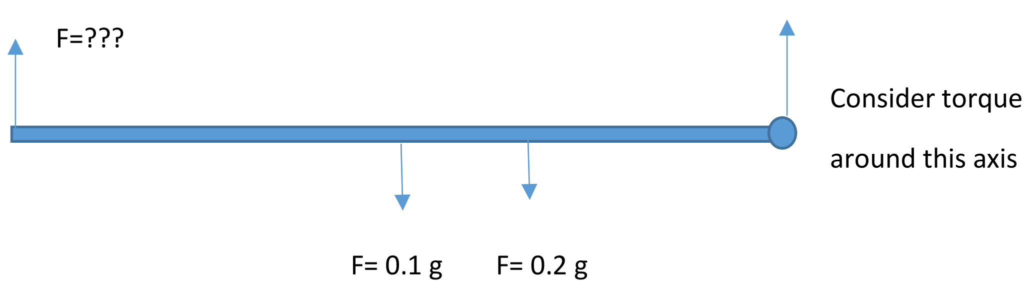
    So, $K/K_0 = 0.5I_1\omega_1^2/0.5I_2\omega_2^2 = (3/4)(4/3)^2 = 4/3$

18. A statics problem, a variation of those done in class. We look at the limiting case where the static friction force is at its maximum and is thus $F_N\mu_s$. By equating forces in the vertical direction, we can see that F_N=200 + W where W is the weight (in Newtons) of the bucket. So the limiting case is where the static friction force, f = 0.5(200 + W) Newtons. Now look horizontally. There are two horizontal forces on the ladder, the static friction force and the force of the wall on the ladder. These must be equal and opposite at equilibrium, so we know that F_W = 0.5(200 + W) Newtons in the positive x direction. Now, for stability, we need to find when the net torque around the point of contact of the ladder and ground is zero. We can choose any axis of potential rotation, but this does seem to be the most obvious. The positive torques are due to the ladder and the bucket, and the negative torque is due to force from the wall. We need to be careful with our sines and cosines, and also need to remember that the mass of the ladder itself acts at its center of mass. In each calculation of the torque we will use force x perpendicular distance (which is the same magnitude as (**r x F**)). We will call the length of the ladder L, but will see that it cancels.



Lcos(60)W + (L/2)cos(60)200 - Lsin(60)(0.5)(200+W) = 0
Now we have an equation with one unknown.
0.5W + 50 = 86.6 + 0.433W
W = 546 Newtons

19. This is a simpler statics question, where we need to make sure that we place force due to gravity of the meter stick at its center. To solve in one go we can take torque around the point of contact of the string and the one meter end of the stick. We then take the total torque and put it to zero, taking the unknown force to be F. All angles are 90^0, so the torque is simply the force times the distance from the axis of rotation.- F x 1.0 + 0.1 x 0.5 + 0.2 x 0.3 = 0 (where 0.5 meters is the distance from the center of mass to the axis, and 0.3 is the distance of the hanging mass from axis) F=1.1 N



The question does not actually say it is horizontal. However, so long as it is a narrow strip, this does not make a difference. If it were not horizontal, there would just be an extra factor on both sides (sinθ) that would cancel.

20. We remember that the angular momentum is defined as **r x p** = (**r)** x (m**v).** So first we need to find v. That is given by $\mathbf{v} = \mathbf{v_0} + \mathbf{a}t$ = 50 – 9.8 x 2 = 30.4 m/s UP.

The magnitude of the angular momentum is therefore 10 x 30.4 x 5 = 1520 kg m/s (as 10 meters is the "perpendicular distance" to the vector of the velocity. Now we have narrowed it down to 2 answers, it is up to the right hand rule to tell us that if **r** is from the observer and pointing east, curl your fingers up for the direction **v**, and the thumb points south. You had lots of practice in class for the right-hand rule, but I suspect some people did not make the direction of the "**r**" vector from the observer to the rocket.