Exam 2 Solutions – Fall 2019

1. There are various ways of doing this, but Work = **F.d** is an easy one and that was shown in the lecture of September 24th. The kinetic energy of the car is $K = 0.5mv^2 = 0.5*1500*20^2 = 300{,}000$ J.
   Work done is the change in kinetic energy. $\Delta K = -300000 =$**F.d** = -F * 25 (because the Force and displacement are in opposite directions. F = 300000/25 = 12000 N

2. Again the work done is the **F.d.** Here, the angle between the two is 37^0, and $\cos(37^0) = 0.8$, so the work done is F*0.8 as the distance covered is 1 meter. This set up is similar to HW-4 number 9. Now to find F. The NET force must be zero for constant velocity, so the horizontal component of the applied force must be equal and opposite to the frictional force $F\cos(37^0) = \mu_k F_N = \mu_k(mg-F\sin(37^0))$, where $\sin(37^0) = 0.6$, and we have taken into account the fact that the normal force is not the same magnitude as the gravitational force.
   F(0.8+0.7*0.6) = 0.7*10*9.8
   F = 56.2N
   Work done is 56.2*0.8 = 45 Joules
   There are a lot of things to get right in this one problem, and it proved to be one the more difficult ones.

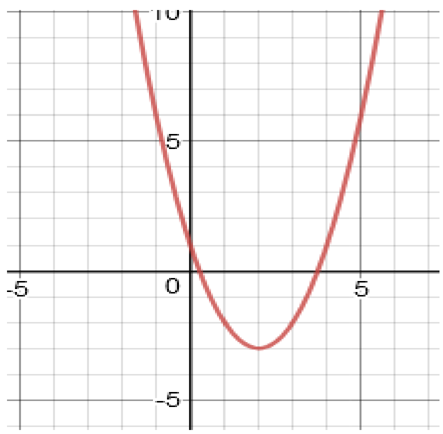
3. The net work done must be the change in the kinetic energy. So this is $0.5*5.0*(10^2-6^2) =$ 160 Joules. September 26th lecture covered an example like this, and it was the question that had the highest success rate.

4. This is the example we discussed at length in class. As you lower it, gravity does (positive) mgh of work, and the spring does negative work. You also do negative work so that it is not moving at its equilibrium point - otherwise it would oscillate around that point. Therefore

   $-mgh < M_h < 0$

5. This relates to the demo in class. The total height lost is $h=4(1-\cos 20^0)$, so the potential energy lost is mgh = 5*9.8*4*(1-.939) = 11.8 J

6. It's always best to sketch a graph. We can see that an object placed at x = 0 has U(x) = 1. It will move to higher x with its maximum kinetic energy being when it is at the bottom of the potential (x=2, y = -3). So at that point it has lost 4J potential energy and must have 4J of kinetic energy. (Even without drawing, I can find the minimum by dU/dx = -4 +2x, so x=2, U(x) = -3.) An example of this was in the class October 3rd.



7. The key point here is that the center-of-mass obeys the regular laws of kinematics and discussed in the lecture of October 5th. The time of the explosion is not important. Where is the "y" center of mass at t=2s?
   $y=15t - 0.5*9.8*t^2$
   $= 15*2 – 0.5*9.8*4 = 10.4$ meters.
   Because one piece is at y=0, the other half is at y=20.8 meters.

8. We need to subtract the velocities taking care to take into account the change in direction (as stressed in an example and demo October 8th) and then convert to a momentum change. Let the original direction be the positive x-direction. Therefore we have $\mathbf{v_1}$ = (40,0),
   $\mathbf{v_2}$ = (-40*cos(30^0), 40*sin30^0), so Δ**v** = (-40(1+0.866), 20) = 77.2, and
   |Δp| = mΔv = 11.6 kg.m/s

9. $0.5m_1v_1^2 = 0.5m_2v_2^2$
   $m_1^2v_1^2 = m_1m_2v_2^2 = (m_1/m_2)\, m_2^2v_2^2$
   $p_1^2 = (m_1/m_2)p_2^2$
   p_1=sqrt(m_1/m_2)

10. Rather simple momentum conservation. The change in momentum of the bullet must be equal and opposite to the change of momentum of the block.
    $M_{bullet}(v/2) = M_{block}(\Delta v)$, and so Δv = v10/(250*2) = v/50

11. This was problem 2 on HW 8, which is problem 10.16 in the text. First specialize the general formula for constant angular acceleration to this problem:

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow 0.75\frac{rad}{s^2}\, t^2$$

Two revolutions is 4π radians which occurs at time $t = \sqrt{4\pi/0.75}\; s \;\cong 4.09\; s.$

12 This was problem 5 on HW 8, which is problem 10.34 in the text. Recall that rotational kinetic energy is $K_{rot} = \frac{1}{2} I\omega^2$. From the graph we read $\omega(4\ s) = 4\ rad/s$ and $(0\ s) = -2\frac{rad}{s} = -\frac{1}{2}\omega(4\ s)$. Hence $K_{rot}(0\ s) = \frac{1}{4}\ K_{rot}(4\ s) = 0.4\ J$.

13. This was problem 9 on HW 8, which is problem 10.56 in the text. The torque (about the pivot point) at point 1 is $\tau_1 = +mgL$ and the torque at point 2 is $\tau_2 = -4mgL$. The moment of inertia is

$I = m[L^2 + (4L)^2 = 17mL^2$. Hence the angular acceleration is $= \frac{\tau_1+\tau_2}{I} = \frac{-3\ g}{17\ L}$. The magnitude of the linear acceleration of point 1 is $a = L|\alpha| = \frac{3}{17}g \cong 0.18g$.

14. This was problem 2 on HW 9, which is problem 11.8 in the text. A solid ball of mass m and radius R has moment of inertia $I = \frac{2}{5}MR^2$. The relation between the ball's linear velocity and its angular velocity is $v = R\omega$, so the rotational kinetic energy is $K_{rot} = \frac{1}{5}Mv^2$ and the total kinetic energy is $K_{tot} = \frac{7}{10}Mv^2$. The potential energy at x = 13 m is 60 J so energy conservation gives $\frac{7}{10}Mv^2 + 60$ J = 75 J and the velocity works out to be $v = \sqrt{\frac{10\times 15\ J}{7\times 0.4\ kg}} \cong 7.32\ m$.

15. This was problem 5 on HW 9, which is problem 11.23 in the text. Torque is

$\vec{\tau} = \vec{r} \times \vec{F} = -\frac{3}{2}\hat{\imath} - 4\hat{\jmath} - \hat{k}$.

16. This was problem 7 on HW 9, which is problem 11.35 in the text. We first compute the force from F=ma. The acceleration vector is $\vec{a} = 8\hat{\imath} - 12\hat{\jmath}$, hence the force is $\vec{F} = 24\hat{\imath} - 36\hat{\jmath}$ and the torque is $\vec{\tau} = \hat{r} \times \hat{F} = 48t\hat{k}$. Note that the time-squared terms in the position are parallel to the force vector, so they drop out and only the linear term contributes.

17. This was problem 10 on HW 9, which is problem 11.69 in the text. The angular velocity of the gyroscope is $\omega = 1000 \times \frac{2\pi}{60}\frac{rad}{s} \cong 104.7\frac{rad}{s}$. You don't need to know the mass of the disk, only the ratio of its moment of inertia to its mass is $\frac{I}{M} = \frac{1}{8}m^2$. Hence the rate of precession is $\Omega = \frac{Mgr}{I\omega} = \frac{8\times 9.8\times 0.055}{104.7}\frac{rad}{s} \cong 0.0412\frac{rad}{s}$.

18. This was problem 3 on HW 10 which is problem 12.28 in the text. The torques about the point A are $\tau_{wire} = +LT\sin\theta = 750$ N-m, $\tau_{bar} = -\frac{1}{2}Lw = -300$ N-m, and $\tau_{block} = -xW$. Because the bar is not moving, the total torque on it must vanish, which implies $x = \frac{450}{300}\ m = 1.5\ m$.

19. This was problem 7 on HW 10, which is problem 12.45 in the text. Recall that the force exerted by an elastic medium if $F = AE\frac{\Delta L}{L}$. The final factor ($\frac{\Delta L}{L}$) is the same for both cylinders but both the area and the Young's modulus of cylinder A are twice as large as those of cylinder B. Hence we have $F_A = 4F_B$. The two cylinders together support the brick's weight $W = F_A + F_B = \frac{5}{4}F_A$, hence $F_A = \frac{4}{5}W$.

20. This was problem 8 on HW 10, which is problem 12.48 in the text. From the figure we read off the Young's modulus as $E = \frac{stress}{strain} = 7 \times 10^{10}$ Pa. The force at displacement ΔL is $F = AE\frac{\Delta L}{L}$. Integrating this gives the work $W = AE\frac{\Delta L^2}{2L} = \frac{1}{2}AE\left(\frac{\Delta L}{L}\right)^2 L = 0.056$ J.